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18 APR 1937
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V. Ramaswami aiyar

Dedicated

To the Memory of

VENKATANARAYANA AIYAR

Ramaswamy Aiyar

FOUNDER OF THE INDIAN MATHEMATICAL SOCIETY

A mind for ever voyaging through strange seas of thought, alone.

14 APR 1937

THE MATHEMATICS STUDENT

Volume IV] SEPTEMBER [Number 3

THE FERMAT POINT OF A THREE-POINT SYSTEM*

BY

V. RAMASWAMI AIYAR.

Let us have n points $A_1\ A_2 \ldots A_n$ which we shall take to be in one plane and n numbers (or "multiples") $\lambda_1\ \lambda_2 \ldots \lambda_n$. We shall call this a "system" and denote it by (A_r, λ_r). Then P being any point, we shall denote by V_P the sum

$$\lambda_1\ PA_1 + \lambda_2\ PA_2 \ldots \ldots \ldots \lambda_n\ PA_n$$

and call this the "Fermat Function" of the point P. Then the position (or positions) of P for which V_P has *the least possible value*, shall be termed the Fermat Point (or Fermat Points). Accordingly if O is a Fermat point of the system, we shall have for any P, $V_P \geqslant V_o$.

The Principle of Composition

If we have two systems (A_i, λ_i) and (B_j, μ_i) for each of which O is a "Fermat Point", we realise at once that O is a Fermat Point of the combined system.

2. *Limitation on* $\sum \lambda$.

If the sum of the multiples $\lambda_1\ \lambda_2 \ldots\ldots \lambda_n$ be negative, it is seen readily that the value of V_P tends to $-\infty$ as P moves to infinity in any direction. The Fermat point of the system therefore occurs at infinity. With this remark, we shall hereafter take $\sum \lambda$ to be $\geqslant 0$.

3. Let a system consist of three points A, B, C with multiples λ, μ, ν. Our object is to fix the Fermat point of this system under all varieties of circumstances.

The principle of composition noted above and a few "Elementary Fermat Point Theorems" enable us to discuss this matter completely.

*[At the invitation of the Madras University V. Ramaswami Aiyar delivered two lectures on the subject of this paper. The lecture notes left by him have enabled the first part of his discourse to be made available to our readers.—ED.]

One and Two-Point Systems

4. If one of the multiples say ν is zero, the system becomes a two-point system $(A, B; \lambda, \mu)$. If two of the multiples say μ, ν are zero, the system becomes a one-point system (A, λ).

ELEMENTARY THEOREM I: Let the system be (A, λ) consisting of a single point A and a positive multiple λ. The Fermat function is λPA which is least when P coincides with A. The Fermat point of the system is the point A itself.

ELEMENTARY THEOREM II: Let the system be $(A, B; \lambda, \lambda)$ where λ is positive. The Fermat function is $\lambda PA + \lambda PB$ which has its least value for points in AB. Hence the Fermat point of the system is the segment AB.

ELEMENTARY THEOREM II (a): Let the system be $(A, B; \lambda, -\lambda)$ where λ is positive. The Fermat point is $\lambda PA - \lambda PB$ which is least for any point of BA produced, *i.e.* if A_i, B_i are the points at infinity in the direction BA, AB, the Fermat point is any point of AA_i (not of BB_i) and A is that extremity of AB to which the positive multiplier λ is attached.

Let the system consist of points A, B with the multipliers λ, μ where $\lambda > \mu > 0$. The Fermat function is $\lambda PA + \mu PB$. This can be broken into $(\lambda - \mu)$ PA and $\mu PA + \mu PB$; or the system may be broken into $(A, \lambda - \mu)$ and $(A, B; \mu, \mu)$. A is a Fermat point of each of the component systems and no other point is so. Hence we see that A is the Fermat point of the whole system.

4·2. Let the system consist of A, B with multiples λ, $-\mu$ where $\lambda > \mu > 0$. The system can be broken into A with positive multiple $\lambda - \mu$ and A, B with multiples μ and $-\mu$. For each component system A is a Fermat point and no other point is so. Accordingly A is the Fermat point of the system.

Three-Point Systems

5. We now proceed to the study of three-point systems $(A, B, C; \lambda, \mu, \nu)$ where none of the multiples is zero. We shall denote the absolute values of the multiples by λ', μ', ν'.

If $\lambda > \mu' + \nu'$ the Fermat point is A. Setting $\lambda = \mu' + \nu' + k$ we have k positive, and the system can be broken into $(A; k)$, $(A, B; \mu', \mu)$, $(A, C; \nu', \nu)$ for each of which A is a Fermat point and no other is so. Hence A is a Fermat point of the system.

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6. We shall next suppose A, B, C to form a triangle ABC, that is, not to be collinear (reserving this case for later consideration).

THEOREM: In the system (A, B, C; λ, μ, ν), if $\lambda = \mu' + \nu'$, the Fermat point is A. For the system breaks into (A, B; μ', μ) and (A, C; ν', ν) for each of which A is a Fermat point and no other is so.

7. Having dealt with the cases where $\lambda \geqslant \mu' + \nu'$ (which discussion includes also the cases where $\mu \geqslant \nu' + \lambda'$ and $\nu \geqslant \lambda' + \mu'$), we are left with the cases where $\lambda' < \mu' + \nu'$, $\mu' < \nu' + \lambda'$, and $\nu' < \lambda' + \mu'$. The multiples λ, μ, ν taking them absolutely, can accordingly be represented by the sides of a triangle. Since the sum of the multiples is positive we have (1) either λ, μ, ν are all positive; or (2), one of them is negative and the remaining two are positive. In the latter case we shall take A to be the point to which the negative multiple corresponds and therefore B, C correspond to the positive multiples; that is we have λ negative, while μ, ν are positive.

We adopt the following procedure. When the multiples are all positive we construct a triangle A′BC on BC, *on the side remote from A*, such that

(7·1) $$\lambda : \mu : \nu = BC : CA' : A'B.$$

When λ is negative and μ, ν are positive, we construct triangle A′BC on BC, *on the same side of BC as A*, such that

$$\lambda' : \mu : \nu = BC : CA' : A'B$$

that is

(7·2) $$\lambda : \mu : \nu = -BC : CA' : A'B$$

Thus, conversely, a point A′ taken on the "off side" of BC will denote a set of positive multiples according to (7·1) above; while if A′ is taken on the "on-side", it will indicate one negative and two positive multiples according to (7·2) above.

With this convention we are able to note the Fermat point corresponding to any position of A′ (that is, the multiples corresponding to that position of A′) most conveniently. We shall call A′, the *indicator* point.

8. We deal first with the cases where the multiples λ, μ, ν are all positive.

Describe the circumcircle of ABC. Produce AB, AC endlessly in these directions to B_a, C_a. Similarly produce BC, to C_b, and CB to B_c. The "off side" of the line BC is now divided into 4 regions, namely

(1) S_1: the segment of the circumcircle bounded by the *chord* BC and the *arc* BC

(2) T_1: The region extending to infinity bounded by the arc BC, and the sides BB_a and CC_a

(3) V_2: The angular region B_a B B_c

(4) V_3: The angular region C_a C C_b.

9. For simplifying our statement it is necessary to introduce a definition.

Definition: Two points P, P′ will be said to be *Companion points with respect to the angle A* of the triangle if P, P′, A are collinear and P, P′, B, C are concyclic. The points P, P′ determine one another.

We require also the following theorem.

ELEMENTARY THEOREM III: The Fermat point of (A, B, C) for the multiples $(-a,\ b,\ c)$ is *any* point of the arc BC of the circumcircle.

The Fermat function in this case is $PB.b + PC.c - PA.a$ which can be shown to be positive, except when P is in the arc BC, (in which case it is equal to zero by Ptolemy's theorem). A proof will be added later on, but let the reader prove this for himself.

10. THEOREM: *If the indicator point A′ lies in the region T_1 the Fermat point is the companion of A′ in the angle A.*

Let F be the companion point of A′, constructed according to the definition in para 9. It can be readily seen that F lies within the triangle ABC and is a point of AA′.

Now the Fermat point of (A′, B, C) for the multiples (– BC, CA′, A′B) is any point the arc BFC of the circumcircle of BA′C (by Elementary Theorem III); and the Fermat point of (A′, A) for the multiples (BC, BC) is any point of the join AA′.

Combining the two systems the point A′ disappears; also F is the only point common to the arc BFC and the join AA′. We deduce that the Fermat point of (A, B, C) for multiples (BC, CA′, A′B) is the point F.

Corollary: Let each angle in the triangle ABC be less than 120°. Let A′BC be the equilateral triangle described in BC on the "off-side". The companion of A′ being F, it is readily seen that each side of the triangle subtends 120° at F. It follows that F thus found is the point for which PA + PB + PC has the least value possible. (This is the original problem proposed by Fermat and is of historic interest.)

11. THEOREM: *If the indicator point A′ lies in the arc BC of the circumcircle of ABC, the Fermat point is A.*

This follows by repeating the argument in § 10. The Fermat point of (A′, B, C) for multiples (− BC, CA′, A′B) is any point of the arc BAC of the circumcircle. The Fermat point of (A′, A) for multiples (BC, BC) is any point of A′A. Combining, A′ disappears and we find A (the common point of arc BAC and line AA′) is the Fermat point.

12. THEOREM: *If the indicator point A′ lies within the circle-segment S_1, then, A is the Fermat point.*

Take A″ on the arc BC such that

$$CA'' : A''B = CA' : A'B.$$

Now take a length l such that

$$CA'' : A''B : l = CA' : A'B : BC.$$

We have CA″ > CA, and A″B > A′B. Hence l > BC.

Let $l = BC + k$; so that k is positive. The Fermat-point for the indicator A′ is that for the multiples (BC, CA′, A′B) that is, for the multiples (l, CA″, A″B) which are proportional to them; that is for the multiples (BC + k, CA″, A″B) that is, for the combined system of

(A, k) and (A, B, C; BC, CA″, A″B):

for each of which A is the Fermat point. Hence it follows A is the Fermat-point of the whole system.

13. THEOREM: *If the indicator point A′ lies in the region V_2, the Fermat point is B.*

We can deduce this from the result in para 12, thus. In our present procedure we have taken the side BC and vertex A, for special treatment and denoted the multiples by an indicator point A′. Had we taken the side CA, and vertex B and proceeded similarly, with an indicator point B′, then, corresponding to A′ lying within the region V_2, the point B′ would fall in the circle segment S_2 bounded by chord CA and arc CA; whence the Fermat point will be B.

Similarly if A′ be in the region V_3 the Fermat point would be C.

14. The reader will note that not only within the region V_2, but for point A′ on its boundary sides, the Fermat point is B. Similarly for points A′ on the boundary sides of V_3, the Fermat point is C. Also for points A′ on the boundaries of the segment S_1 the Fermat point is A.

We have now completely dealt with the question of the Fermat point of points A, B, C (forming a triangle ABC) for any system of *positive* multiples. It remains to discuss the Fermat point when one of the multiples is negative and the remaining two, are positive.

(*To be continued.*)

[The author's notes terminate here and would seem to correspond roughly to the first discourse. According to the reports of the lectures, the second day's discourse was devoted to an analytical discussion of the problem and the explana ion of the absence of Fermat points from certain regions.—ED.]

NOTE ON A CLASS OF CURVES*

BY

V. RAMASWAMI AIYAR

I wish to draw attention to a class of Curves (class, in the *general* sense) which has some analogy, and relation, to the class of curves considered by Clifford in evolving his well-known series of what may be called the Clifford-Miquel theorems, where for every $2n$ lines we get a point, and for every $2n-1$ lines a circle ($n=2, 3, \ldots$).

2. The kind of curves considered by Clifford are of class n, and touch the line at infinity $(n-1)$ times; and can be determined individually when $2n$ tangents of the curve are given. We shall speak of such a curve as a C_{2n}.

Every C_{2n} has *a unique focus.* If any $2n-1$ tangents of the curve be taken the Clifford-Miquel circle of the $2n-1$ lines passes through the focus. If $2n$ tangents be taken, the focus becomes the common intersection of the $2n$ Clifford-Miquel circles—of the lines taken $2n-1$ at a time. That is, the focus is the Clifford-Miquel point of the $2n$ lines. When $n=2$, we have the well-known properties of a parabola.

*[This is the last paper received from Ram swami Aiyar for publication in the Student. In a letter written to me in August 1935, he wrote " How I wish 42 years ago when I took the M.A. degree, somebody had presented me with a copy of Clifford's *Mathematical Papers.* I got a copy of the book from our library at Poona for reference. I wish I were young again to follow up some enquiries on the lines of Clifford ! But on 4-8-35 I have got into the 65th year of age. However, I can see and admire the work of young India, which has greatly improved since those days......... "—ED.]

3. The class of curves that I desire to bring to the notice of the reader is also one that can be determined individually when $2n$ tangents are given.

It is of class $n+1$; and touches the line at infinity n times,—*the circular points at infinity being always two of the points of contact.* As already stated it can be determined when $2n$ tangents are assigned. Let us refer to such a curve as an R_{2n}.

The property of an R_{2n} is:—

If any $2n+1$ tangents of an R_{2n} be taken, the Clifford-Miquel circle of the $2n+1$ tangents becomes A STRAIGHT LINE. I found this recently.

When $n=2$, the R_{2n} becomes a three-cusped hypocycloid; and theorem becomes:—

If any five lines are tangents to a three-cusped hypocycloid, their Miquel circle becomes a straight line.

This property was given by me in Question 1250 of the journal of our Society. [See volume for 1922, p. 159].

V. RAMASWAMI AIYAR.

RAMASWAMI AIYAR ON PEDAL TETRADS OF A CONIC

BY

R. VAIDYANATHASWAMY

On looking over my old papers, I find a letter dated 14-2-1931, from Chittoor, from the late Mr. V. Ramaswami Aiyar relating to my paper "On the Feet of Concurrent Normals of a Conic" (J. I. M. S., Vol. 18, 1930). His observations on the results of the paper are characteristic of the quickness of his mind, in grasping the various aspects of a geometrical problem and in suggesting new approaches and relationships. As his remarks are likely to be of interest to the general reader, I reproduce his letter.

Chittoor, 14-2-1931.

My dear Vaidyanathaswami,

"...... Nevertheless I read it with great interest on account of several neat theorems that you have given which are intelligible to

the old view-point and quite arrest one's attention by their neatness. I found myself able to prove many of them for myself by recalling theorems I knew, or newly working out some in the old way, and then putting "two and two together" for getting at your theorems—that one feels enriched in mind for knowing. I send you some of these theorems that were of help in understanding yours with some comments on these latter.

I

(1) The axes of parabolas circumscribing a triangle ABC are pedal lines of the *medial triangle.*

(2) The axes of parabolas inscribed to a triangle ABC are pedal lines of the *anti-medial triangle.*

(3) Any parabola circumscribing a triangle ABC can, by a translation parallel to its axis, be changed into one inscribed to its second medial triangle".

II

"Considering triangles ABC inscribed in a circle X and circumscribed to a parabola Z whose focus S lies on the circle we have:

(1) $SA \cdot SB \cdot SC = \text{const} = 4Rp$ where p is the parameter (focal vertex distance) of Z, and R is the radius of X;

(2) The in and excentres of all such triangles lie on a rectangular hyperbola H (that is, the variable triangle ABC is self polar with respect to H);

(3) The centre of H is S;

(4) The asymptotes of H are the bisectors of the angles at S contained between the axis of Z and the tangent to X;

(5) If α be the semi axis major of H

$$\alpha^2 = \frac{SA \cdot SB \cdot SC}{2R} = 2Rp, \text{ by (1)}"$$

III

"If the axes of three parabolas, foci S_1, S_2, S_3, inscribed to a triangle ABC are concurrent at K, then K is the orthocentre of $S_1 S_2 S_3$. (Mere rehash of your (21))

Observations

(1) We get your points p, q, r as the points in the arcs BC, CA, AB of the circumcircle for which the product of the distances to the vertices is a maximum.

(2) Your Theorem (13) was that which struck me first by its simplicity and novelty. If ABC be a pedal tetrad on a parabola, its circumcircle passes through V the vertex, and its centroid G lies on the axis. The axis being always a pedal line of the medial triangle, taking it as a line in the medial triangle, the corresponding line of ABC will be a pedal line of ABC; but this corresponding line of ABC is the axis itself as it passes through G.

(3) In your Theorem (12) it is worth observing that the parameter of the parabola S′ is four times that of S. We have the theorems:

If two parabolas S, S′ are co-axal (and turned the same way) and the parameter of S′ is 4 times that of S; then

(*a*) triangles ABC can be inscribed in S so as to circumscribe S′,

(*b*) their circumcircles pass through the vertex of S and focus of S′,

(*c*) they have a fixed centroid G,

(*d*) the vertices of ABC form pedal triads on S and the points of contact of the sides with S′ form pedal triads on S′.

[These are only a rehash of your theorem (12)]

(4) I obtained your theorem (24) differently and then on reading your own proofs, am very much struck by them. I am also struck with your theorem (20) and the way you use it for proving (24).

(5) As regards your construction for the point of concurrence of normals in the case of the three maximum circumparabolas of ABC, I am afraid it is a case of Homer nodding.* From Casey's equation in page 185 of his *Conics* for the Joachimsthal circle of a point α, β with respect to $y^2 = 4ax$, I form the following construction. Let p be the focus of one of the max. inscribed parabolas in question.

Let p G cut the circumcircle in V. This will be the vertex of the corresponding max. circumparabola.

In VG produced take V′ so that $VV' = \frac{3}{2}$ VG.

Bisect V′p in g.

* The construction referred to occurs in the closing lines of my paper and is erroneous. R. V.

Through p' (diametrically opposite to p) draw $p'p''$ parallel to the axis.

Then the reflexion of g in $p'p''$ is the point of concurrence required.

I get this from Casey's equation referred to and the rehash of your theorem (12) that I have given above.

Kindly examine the matter and give me the result of your consideration.

(6) As regards the pedal triads in a conic S, I knew that they could be exhibited as forming an infinity of what may be termed Steiner-Joachimsthal systems. *A Steiner-Joachimsthal system* will consist of the triangles inscribed in S and circumscribed to a parabola touching the axes of S so that the chord of contact is a tangent to S. Variation of the tangent gives the infinity of such systems.

I owe to you the interesting fact that they can also be classed into an infinity of what may be fitly termed *Burnside-Vaidyanathaswami* systems.

A system of this kind will consist of the triangles inscribed in S and circumscribed to a co-axal conic S′.

[I note that the conics S′ are those which are such that the tangents at the extremities of their axes *form a rectangle inscribed in the rhombus* formed by joining the extremities of the axes of S.]

Kindly enlighten me on the following point. If triangles pqr inscribed in a conic S and circumscribed to one S′ mark pedal tetrads on S then, *is* the system necessarily a "Steiner-Joachimsthal" one or, failing it, a "Burnside-Vaidyanathaswami" one? [I expect the answer to be 'yes']" With kind regards.

Yours sincerely,

V. RAMASWAMI AIYAR.

V. RAMASWAMI AIYAR, THE FOUNDER OF THE SOCIETY

A LIFE SKETCH*

BY

M. T. NARANIENGAR, M.A., *Bangalore*

"Professor" V. Ramaswami Aiyar was born on the 4th August 1871 at Satyamangalam, in the Coimbatore District, in his maternal grandfather's house. His paternal grandfather Ramaswami Aiyar, known also as Munshi Ramaswami Aiyar from the fact that he held the important post of 'Munshi' under the District Collector of Coimbatore for a long time, was a good Sanskrit Scholar and was noted for his piety and generosity. He became a Tahsildar in course of time and commanded great respect in his Taluk.

V. Ramaswami Aiyar's father died when the boy was barely six months old and one of his paternal uncles brought him up and educated him in the local College to the end of the F. A. Course. During the B. A. and M. A. Courses at the Presidency College, Madras, he was supported by another of his paternal uncles who was a Tahsildar, by name R. Ramachandra Aiyar.

It is noteworthy that Abbu—that was the pet family name of our Professor—did not go through the formality of *aksharabhyasam*, as is the Hindu custom in all orthodox families. When he was four years old, he was missing from the house one day, and after an anxious search, he was traced to a neighbouring *pial* school, with a broken chip of slate. This is how his informal Schooling began and shows his thirst for learning at that early age of four. From his school days, his habit of deep concentration was characteristic of him. He was often found poring over his books, while his companions were engaged in play and amusing themselves. Throughout his scholastic career, he was the 'top boy' in his class and carried away the First Prize, from the First Standard right up to the F. A.

During his undergraduate course in the Coimbatore College, Mr. R. Krishna Aiyar, B.A., another of his uncles, was Mathematics Lecturer, and Vice-Principal of the College. The neatness and

elegance with which V. Ramaswami solved problems set in class evoked his uncle's admiration.

Unlike the present-day school boys, Ramaswami Aiyar had a strong physical constitution which stood him in good stead throughout his scholastic career. He took an active part in cricket and gymnastics from his boyhood. The systematic practice he had in these games in the Coimbatore Literary Students' Association—of which he was an Organiser—enabled him to carry off several Sports Prizes annually at the College Sports. About this period, he won the First Prize in the Competition held at the Madras Peoples Park Exhibition.

His Literary and Artistic tastes induced him to take part in the dramatic performances in connection with the anniversaries of the Students' Literary Association.

While undergoing the graduate course at the Presidency College, his contributions to the Educational Times and other Mathematical Journals, which were remarkable for their originality and elegance, attracted the attention of his Professors. The Editor of the Educational Times addressed him as "Professor Ramaswami", judging from his contributions that he should be a College Professor. This title of 'Professor' stuck to him ever afterwards.

He graduated from the Presidency College with distinction in the year 1890, and soon afterwards he was offered a clerkship in the Collectorate, as a mark of appreciation of his distinguished performance in the B. A. Degree Examination. This glorious offer of Rs. 15 per mensem, he was forced to accept owing to family straitened circumstances.

At this juncture, his Professors, Rai Bahadur Ranganadham Modaliyar, and G. H. Stuart took an interest in his career and secured for him a Scholarship to prosecute his studies for the M. A. Degree at the Presidency College. He gladly threw up the clerkship at the Collector's Office and entered on the M. A. Course. His success in the M. A. Degree Examination in 1893 proved a stepping-stone for him in his subsequent career.

He sat for the Mysore Civil Service Examination along with his friend C. Hanumanta Rao, another distinguished graduate of the Madras University; both however failed to come within the rank.

About this time, Professor Venkataswami Nayudu of the Central College, Bangalore, fell ill and had to go on long leave. In the

acting arrangements during Mr. Nayudu's period of absence, Professor Ramaswami got his chance. Sir K. Seshadri Aiyar, Dewan of Mysore who had heard of Ramaswami's high attainments in mathematics, sent for him and offered him the Acting appointment, which he filled with credit. When the temporary appointment ceased on the return of Professor Venkataswami Nayudu from leave, Mr. Ramaswami Aiyar was appointed to the Mysore Maharajah's College to assist Principal Weir in teaching Mathematics for the B. A. Degree. During his stay in Mysore, his close association with Mr. Weir, who was a Cambridge Wrangler, enabled him to secure memberships of various Mathematical Societies of established reputation. His contributions to the Journals of these Societies won for him distinction as a mathematician of high eminence.

In 1898, he sat for the Madras Provincial Civil Service Competitive Examination at the earnest solicitation of his friends and relations and was successful in the Test. Much against his inclination, he had to give up the Teaching Profession which he loved so much and join the Madras Provincial Service. After holding some non-gazetted appointments during the probationary period, he started career as Deputy Collector in 1901. He retired from Service in 1926 after 25 years of work in various capacities, on a well-earned pension. Thereafter he made Chittoor his home and led a peaceful and contented life there, devoting all his leisure to his favourite subject.

Throughout his official life as Deputy Collector, his interest in mathematics never flagged. Stories are rife as to his absent-mindedness while in the midst of trials of Criminal Cases, as a Magistrate. It appears that he would often be found lost in tackling the solution of a mathematical problem, completely forgetting the Case before him, and after a time, as if awaking from a dream, would ask the Pleader to start his arguments afresh. This absent-mindedness, or abstraction, was a habit with him even in his domestic life. When at dinner in the company of friends and relatives, he was often discovered lost in thought; he would abruptly get away without finishing his dinner, and would go to the table to note down a solution or a problem which had been engaging his attention all the time.

While he was Deputy Collector at Gooty in 1906, his love of mathematics made him feel keenly the lack of facilities for advanced study and research in this country. He set to work to organize an Association for removing these disabilities. He addressed a few friends interested in mathematics, suggesting the formation of an Association called the Analytic Club, to secure facilities for Study

and Research, by subscribing for Mathematical Journals and by purchasing books for Advanced Studies. On the assurance of his friends that they would co-operate with him in the laudable object, he issued his 'famous' Circular Letter, dated 25th December 1906, in which the following bold announcement of his Scheme is found:—'I propose to consider the Club formed as soon as *three friends* have agreed to the proposal, making with me *four members*.' Three months later, he announced in the Madras Dailies, the formation of the Indian Mathematical Society, with 20 members, on the 4th April 1907. Poona was chosen as the Headquarters of the Society and he himself acted as its Secretary till 1910. Meanwhile, he started the Society's Journal in its useful career and established the Library. His capacity for Organization is well demonstrated by his labours in Founding the Indian Mathematical Society and framing its Constitution. The bundles of cyclostyled Circulars which he issued in the capacity of Secretary during his tenure of Office go to prove the strenuousness with which he laboured for the Society in the early days of its life. He withdrew from the Committee in 1910, after satisfying himself that the Society was firmly established. He did not enter the Committee again, except as President when in 1926 an Address was presented to him at the Bangalore Conference. He continued to be the President till 1930. He attended all the Conferences of the Society except one, and actively participated in their deliberations. His last trip to Delhi, in spite of the advice of friends to the contrary, in connection with the Conference held there, proved too trying for him, perhaps. Though he was seemingly all right, he took ill all of a sudden on the 20th January 1936. Cerebral haemorrhage followed and he expired on the 22nd January 1936 at 3-30 P.M., in spite of the best medical treatment, at Chittoor.

A few days before his death, when he was reviewing his life, dividing it into eight Octaves, in his usual characteristic manner, he said he was proud of two things done in his life, both of which occurred in the fifth Octave—the Prime of his life. One of them was the discovery of the renowned Mathematical Genius, the late S. Ramanujan, F.R.S., and the other the Founding of the Indian Mathematical Society.

The story of the discovery of Ramanujan is worth repeating here, though it has found a place in the Obituary Notices of Ramanujan's life in our Journal.

When Ramaswami Aiyar was Deputy Collector in Tirukkoilur in 1910, Ramanujan sought his patronage and asked to be appointed

as a clerk in his Office. Ramanujan's only recommendation was his 'Ponderous' Note-Book, which has now become famous and is preserved in the Madras University Library. It was little known at that time that that Note-Book contained anything of value. On perusing the contents of the note-book, Ramaswami Aiyar was struck by the extraordinary results recorded therein, and declining to smother his genius by an appointment in his department, recommended him to his mathematical friends in Madras holding high positions. The story is well-known how eventually Ramanujan was made a Research Student of the Madras University, was deputed to the Cambridge University and achieved distinction as the Greatest Indian Mathematician!

No one has done so much to inspire enthusiasm for mathematical Research in India as Prof. Ramaswami. He may be said to have created the present 'craze' for original work in our land. The Journal founded by him has been responsible for discovering many a talented mathematician of the country, who would otherwise have gone unnoticed and unheard. His eyes used to glow with affection and delight whenever he met a student who had contributed to the Journal. Nothing gave him greater pleasure than to be surrounded by promising young mathematicians to whom he would go on talking mathematics endlessly. Many a young man will remember with gratitude the remarkable way in which they were inspired by his writings and talks and above all by his unique personality. One of his high ambitions in life was to deliver Lectures in Mathematics, under the auspices of Universities. Thanks to the present Vice-Chancellor and the Syndicate of the Madras University, he was given an opportunity to do so in November last, and he was proud of the same.

At the Silver Jubilee Celebration of the Indian Mathematical Society held in Poona on the 26th March 1932, under the Presidency of Dr. Paranjpye, the following Resolution was adopted:

'The Society offers its warmest and most felicitous Congratulations to Mr. V. Ramaswami Aiyar, the Founder of the Society and its most ardent and most enthusiastic Organizer, on the occasion of the completion of 25 years of its most useful work in the cause of Higher Mathematics in India.'

In private life, Ramaswami Aiyar was a loving husband and an affectionate father. Though endowed with a large family, his absorption in mathematics never allowed him much leisure to attend to

domestic affairs. His devoted and loving mother came to his relief by looking after his family. Having married early, he was a father when he was a student. He lost his first wife when he was at Atmakur in 1901 and married a second time in 1902. He has left behind him, his wife, three sons and four daughters, a large number of grandchildren, numerous relations, and a very wide circle of friends, to bemoan his loss. He was a perfect gentleman, and a friend of the poor and depressed. His childish simplicity struck every one with whom he came in contact. This trait in his character was appreciated mostly by the children, who recognised in him an affectionate 'grandpa'. He was good at story-telling and it was this quality and his unspoilt intuition that attracted children towards him. It was this same intuition that enabled him to 'revel' in mathematics.

As a boy, he never quarrelled with any of his mates and had an unruffled temper which stood him in good stead in his official life. He did thoroughly everything he undertook, and his Official Career was characterised by sturdy independence and uprightness.

He used to confess to his intimate friends that he made a mistake in having resigned his appointment in the Mysore College to enter the Madras Civil Service. Had he continued in the Mysore Service, he would have brought credit undoubtedly to the Mysore University. No one deserved the appellation of 'Professor' better than he, and his reputation as Professor would have been doubled had he stuck to the Teaching Profession.

The Society has lost in Professor Ramaswami a 'Tower of Strength' and All-India is poorer to-day for the loss of such an enthusiastic and gifted Worker. May his Soul rest in Peace!

[*This Life Sketch is based on Notes kindly supplied to me by Professor Ramaswami's intimate friends and relations Messrs. G. A. Srinivasan, Vaideeswara Aiyar, and on Mr. S. R. Ranganathan's Article in the Hindu, which appeared soon after the Professor died.—M. T. N.]

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TRIBUTES AND PERSONAL REMINISCENCES

FROM DR. R. P. PARANJPYE, VICE-CHANCELLOR, LUCKNOW UNIVERSITY, AND PRESIDENT OF THE INDIAN MATHEMATICAL SOCIETY.

The Indian Mathematical Society has suffered a grievous loss by the death of its founder Mr. V. Ramaswami Aiyar. I was in close touch with him when he first conceived the idea and he carried on an extensive correspondence with Mathematicians in all parts of India regarding it. His first idea was to make mathematical literature in the form of journals and advanced books available to all interested in the subject and he laid down the first lines of procedure in circulating various journals among the members. The library was located at the Ferguson College, Poona, and I was the honorary librarian though most of the actual work was done by my colleague the late Prof. V. B. Naik. Soon he started the publication of a progress report which, besides giving news about the affairs of the Society, contained mathematical notes received from members. Within a year or two this was transformed into the Journal of the Indian Mathematical Society and now appears in two separate sections, one dealing with advanced, and the other with elementary portions of the subject. The foundation of the Society and its subsequent developments are practically due entirely to Ramaswami's energy and enthusiasm.

He was a member of the Provincial Civil Service in Madras and held important administrative positions. His heart, however, was in Mathematics and over and above organising the Society he made frequent contributions to the Journal. His Mathematical attainments were of a very high order and one may be excused the feeling that had it been possible for him to be appointed a professor of Mathematics in the Indian Educational Service he would have turned out more original work. It is interesting to see that among those who entered enthusiastically into his scheme for the foundation of the society there were two other high officials in Government Service viz. Dewan Bahadur R. Ramachandra Rao and Mr. Balak Ram, I.C.S., of whom the same can be said. But it is useless to speculate on the might-have-been. We can only thankfully acknowledge what they did for the Society and for the advancement of Mathematics. By Ramaswami's death the Society has lost its father and its moving spirit and many of us a valued personal friend. In spite of the long

distances in India Ramaswami attended several of the sessions of the Society and took a prominent part in the administration and technical discussions. His enthusiasm was contagious and everybody came to have a personal feeling of regard for him.

FROM SIR V. RAMESAM, HIGH COURT JUDGE (RETD.), MADRAS,

I first made the aquaintance of Mr. Ramaswami Aiyar in the year 1890. Early in that year, he had taken his degree and had ceased to be a regular student of the Madras Presidency College. I joined the College soon after he left it. In those days students studying for the M.A. degree were permitted by the College authorities to use the College library for a small fee and Mr. Ramaswami Aiyar availed himself of this concession. In this we came into contact and the aquaintance so begun ripened into a friendship till it had terminated by the cruel hand of death.

From the beginning, his mind showed an original or constructive bent of mind in his Mathematical studies. In studying text-books he would anticipate later theorems and very often, he would discover theorems which, to his disappointment are to be found in the succeeding paragraphs or chapters. In the matter of solving examples, the usual type of exercises at the end of the chapters did not generally interest him. Unless a question promised some interesting result or incited some elegant device for its solution, he did not care for it. Accordingly he was doing more constructive work, by way of proposing problems for solution rather than training himself in the solution of questions. Even in the examinations, he would leave questions untouched which men of inferior calibre would have solved. At that time, the Indian Mathematical Society was not in existence and he was sending his contributions to the Educational Times, in which he was described as Prof. V. R. Aiyar though he was not then a professor in any college. One of his earliest contributions is his proof of Prof. Genese's Theorem—(Q. 10879).

If P and Q are two isogonal conjugates in a triangle and S is its circumcentre, $(R^2 - SP^2)(R^2 - SQ^2) = 4R^2\alpha\alpha' = 4R^2q^2$ where α and α' are the perpendiculars from P and Q on any side.

Two solutions of this theorem—one by Mr. Ramasami Aiyar—appeared in the Educational Times. (Vol. LV, p. 97 and Vol. LVII, p. 37). (Vide Gallatly's *Modern Geometry of the Triangle* 2nd edition p. 76.)

In 1893, he took his M.A. degree. Soon after, he acted as professor of Mathematics in the Central College, Bangalore, for Professor

T. Venkataswami Naidu. He thus formed a lasting friendship with him and Prof. M. T. Naranaiengar both of whom collaborated with him in the founding of the Indian Mathematical Society. He had high respect for their mathematical abilities. In 1895 he became a permanent professor in the Maharajah's College, Mysore, and, on the recommendation of its principal, Mr. Weir, became a member of the Edinburgh Mathematical Society. This immediately bore fruit. In Vol. XV of the *Proceedings of the Edinburgh Society* p. 75 he gave us an extension of Prof. M'Cay's theorem which itself was an extension of Fuerbach's Theorem.

Ramasami Aiyar first proved the following theorem:—

If P and Q are isogonal conjugates, M the mid-point of PQ and N the nine-point centre, then $SP \cdot SQ = 2R \cdot MN$.

Combine this with Prof. Genese's Theorem,

$$(R^2 - SP^2)(R^2 - SQ^2) = 4R^2q^2.$$

$$R^4 - R^2(SP^2 + SQ^2 + 4q^2) + SP^2 \cdot SQ^2 = 0$$

$$\therefore \quad R^4 - R^2(SP^2 + SQ^2 + 4q^2) + 4R^2 \cdot MN^2 = 0$$

$$\therefore \quad R^2 - (SP^2 + SQ^2 + 4q^2) + 4MN^2 = 0$$

$$\therefore \quad R^2 + 4MN^2 = SP^2 + SQ^2 + 4q^2$$

$$\text{Now} \quad q^2 + \left(\frac{SP \pm SQ}{2}\right)^2 = \tfrac{1}{4}(SP^2 + SQ^2 + 4q^2) \pm \tfrac{1}{2}\, SP \cdot SQ$$

$$= \tfrac{1}{4}\,(R^2 + 4MN^2) \pm R \cdot NQ$$

$$= (\tfrac{1}{2}\, R \pm MN)^2.$$

The circle centre M whose radius is ρ where $\rho^2 = q^2 + \left(\frac{SP \pm SQ}{2}\right)^2$ touches the nine points circle, and this circle is the circle passing through the points of intersection of perpendicular tangents, one to the conic inscribed in the triangle with P and Q as foci and the other to its confocal passing through S.

The first theorem $SP \cdot SQ = 2R \cdot MN$ is referred to as Mr. Aiyar's theorem in Gallatly's *Modern Geometry* (p. 79) and there is another proof by Mr. Gallatly in *E. T.* XVII, 29.

While teaching his pupil, Miss Bhabha (afterwards Lady Dorab Tata), a simple proof of the extension of the Pythogorean theorem to a polygon of n-sides suggested itself to Mr. Ramasami Aiyar. (Vide *Math. Gaz.* Vol. I, 109). In 1895 he passed the Provincial Civil Service Examination and became a Deputy Collector.

In 1901, he made a fresh extension of Feuerbach's theorem and intended to publish a paper but it was never finished. Particular cases of it appeared as questions in the *E. T.* in 1906 and 1907.

Though Ramasami Aiyar was at Cuddapah and Prof. Naraniengar was at Bangalore, we have got two proofs of Feuerbach's theorem from them, which are practically identical and differ only in a minor detail.

Even before Ramasami Aiyar's proof the simplest geometrical proof is Prof. M'Cay's proof (Mclellend's *Geometry of the Circle* p. 201) and not, as Prof. Nexon would say, the proof by Prof. Purser in *Euclid Revised*.

About this time he founded the Indian Mathematical Society for an account of which one may refer to his address in Vol. XVI Suppt.

In Vol. IV we have got one important paper on the Orthopole. This paper was stimulated by Lemoyne's theorem—"the pedal circles of points lying on a line cut a fixed circle orthogonally". One important theorem in it is the following.

If X be any conic inscribed in the triangle ABC and Y a confocal to it touching ZZ′, then the mutual orthoptic circle of X and Y cuts the orthopolar circle of ZZ′ orthogonally. The theorem in the *Proceedings of the Edinburgh Math. Society* follows from this immediately as shown by Mr. A. A. Krishnasami Ayyangar (*Math. Gaz.* Vol. XV 258). But Prof. Rogers rediscovered it and gives an analytical proof *Math. Gaz.* Vol. XV. 111). He is unaware of the fact that Ramasami Aiyar found it in 1895. From the many questions and papers given by him, we see that Geometry was the *forte* of Ramasami Aiyar as he himself thought. (Vol. XVI Suppt.)

The last contribution by him was a lecture on the Fermat point* delivered in the Senate House. On the second day I happened to preside and concluded my speech with the hope that his life will be long spared in the interests of Mathematical Science. Alas! the hope was immediately dashed to the ground.

FROM DR. RAM BEHARI, UNIVERSITY OF DELHI, HON. JOINT SECRETARY OF THE INDIAN MATHEMATICAL SOCIETY.

I greatly value the opportunity, so kindly given to me by the Editor of *The Mathematics Student*, to pay my last tributes of regard and affection to V. Ramaswamy Aiyar, the founder of the Indian

* Vide pp. 101–106 of this issue of the STUDENT.

Mathematical Society, who passed away in Chittoor on the 22nd January 1936.

The news of his death came as a shock to us in Delhi, for just about a month before, we had him in our midst, full of life and cheer, untiring in his endeavour to pursue his ideas and ideals, devoted to Mathematics and to the Indian Mathematical Society of which he was justly proud of being the founder. The news was received with profound grief by all those who had come to know him during his short stay in Delhi.

I first met Mr. Aiyar in December 1932 at Bombay on the occasion of the Eighth Conference of the Society, but it was at the Delhi Conference, three years later, that being the local Secretary, I had the privilege of coming into close contact with him and knowing him intimately.

Although Mr. Aiyar was suffering from ear-ache when he arrived in Delhi and was not free from pain during his stay here, he was as energetic and cheerful as ever. He was a man of clear ideas, and he held them with a tenacity and intensity of conviction which never let him rest till he had secured their acceptance by others. The passionate earnestness of his convictions gave him a great driving power which in the end usually proved irresistible. He had a dynamic and dominating personality. Nevertheless, his respect for the personality of others was as great as it was genuine. He was gifted by God with some extraordinary qualities of head and heart which endeared him to all those who happened to come in contact with him.

Two things stand out in my memory of V. Ramaswamy Aiyar: his intense love for mathematics and the Indian Mathematical Society and his unceasing quest for knowledge, experience and Truth.

In spite of his old age, his mind was astonishingly alert and active, and he seemed to possess a vast store of intellectual energy. During the conference days he particularly loved to spend his spare moments, discussing mathematical problems with mathematicians of the younger generation. As an example of his absorbing interest in mathematical discussions, I may give a little incident of the last Mathematical Conference. One day as he was expounding some of his neat results on limits and inequalities, he inadvertently broke his wrist watch while thumping the table before him, and at the close of his discourse, wondered how the glass and hands of his newly bought watch happened to be missing all at once.

Nor was his life devoid of interesting human touches. He used to relate with great delight how happy he was when a well-known English mathematician addressed him in a letter as "Professor" Ramaswamy Aiyar, when the chair he sat in was that of a Deputy Collector.

He was always eager to impart to others the knowledge he possessed. To use his own words "Human mind is like a well, the more water you draw out from it the fresher it becomes, otherwise it stagnates". In his pleasant and inspiring company, younger mathematicians forgot all other worries and felt an inward urge to devote themselves wholeheartedly to the cause of Mathematics, finding pleasure in the very pursuit of knowledge.

At the age of 65, in spite of a severe ear-ache he undertook all the hardships of a tedious journey from Chittoor to Delhi, took an active part in all functions, and read two valuable papers before the Conference.

His eagerness to acquire knowledge and experience was remarkable. After the conference, when other delegates had left, Ramaswamy Aiyar decided to stay behind for a few days to see—not with the casual eye of the tourist but with the reverend interest of a learner—all the important historical places in Delhi. While seeing the Old Fort and Asoka's pillar, he repeatedly remarked with great joy, "I am certainly luckier than my young friends who did not have this opportunity". When shown from a distance the famous library room on the staircase of which Humayun slipped to meet his death, Ramaswamy Aiyar insisted that having come so far he would see with his own eyes the exact spot at which the great emperor slipped, an event of which he had read when he was a small boy at school. He did go right up to the marked stair and standing there humorously remarked to me "A heavy old man as I am, I can imagine much better than you that by slipping down from this stair, poor old Humayun could not escape death."

During the short stay in Delhi, in spite of his extremely busy programme, he went through the narrow, crowded bye-lanes of Delhi to have a *darshan* of Shanti Devi a nine-year-old girl, who was alleged to remember and relate all the incidents of her previous life, and referred subsequently to this visit in very happy terms.

He believed that knowledge and experience gathered by the soul in one life were not lost at death, but continued, in a nascent form, in the next life. Thus intellectually—and more generally speaking,

spiritually—man should strive unceasingly after knowledge and experience in this life in full confidence and faith that this effort of his is not without meaning in the course of a long unending process.

In the death of V. Ramaswamy Aiyar, the Society has suffered an irreparable loss. The most fitting manner in which mathematicians in India can perpetuate his memory, is to promote the Society which he founded, carefully nursed, and loved so dearly. His life was an example of courage, perseverance and devotion to the cause of Mathematics. He will be remembered with regard and affection by a large circle of friends and all lovers of mathematics.

FROM PROF. D. D. KAPADIA (RETD.), POONA

I first came to make Ramaswami Aiyar's acquaintance in April 1907, when he approached me with a request to join the Indian Mathematical Club. Without knowing each other by sight for more than nine years, we became such good friends, that nearly every year we exchanged birthday greetings. Even last year while I was out of India, I received his greetings, but I never in the least dreamt, that it was the last I was to receive from his loving hands!

After nine years of work, when we knew one another simply by correspondence, we had the unique pleasure to meet physically and to solidify our acquaintance at the First Conference of our Society, held at Madras in December 1916. It was a unique occasion and a unique experience and the memory of the happy pleasurable days spent during the Conference at Madras are still fresh in my mind.

An idea of what voluminous work was undertaken and achieved by him, during the first four years of the existence of our Society, can easily be obtained by casting a glance at the files of the Honorary Secretary during those days, and it is really remarkable, how he was able to do all this in addition to his multifarious official duties! But such a state of affairs could not continue long, and he had to give up his Secretaryship in 1910, though his interest in the affairs of the Society remained unabated till the last day of his life. He was really a great genius, and it was highly regrettable that instead of being attached to the Educational Department, he was obliged to work in the Revenue Department. There is not the least doubt, that if he had continued in the educational Department, the life history would have been sung in quite a different tune. I had a

very great regard for him and it shall ever remain fresh in my heart. May his soul rest in peace and ever send his choicest blessings to our Society! Amen.

FROM PROF. A. A. KRISHNASWAMI AYYANGAR,
MAHARAJA'S COLLEGE, MYSORE.

One sunny afternoon twenty years ago I saw a genial figure of generous build seated on the floor of an upstair room in Madras, amidst a fairly good gathering of ardent disciples who were listening intently to a conundrum from the lips of the master. The First Conference of the Indian Mathematical Society was then taking place in the Presidency College buildings, Madras, and the party that I met was apparently taking rest after a heavy mathematical dose at the morning conference and preparing for the evening social functions. The conundrum interested me and I immediately made a note of it and its solution in my note-book. This was my first direct introduction to the great Founder of the Indian Mathematical Society and his mathematics. The acquaintance and the vision were a great inspiration to me as they were to many others who associated with him. Those who felt bored by the dry bones of mathematics, a *darsan* of the late V. Ramaswami Aiyar would have convinced how mathematics could pulsate with so much life and blood. He loved not only mathematics but all mathematicians. Especially the younger generation should be highly indebted to him for the cheerful and encouraging mode in which they were egged on to research by his fascinating mathematical talks full of spicy anecdotes. He was always full of the mathematician's joy and a few minutes' association was enough to catch the infection from him.

The popular adage 'One mathematician more, one man less' may well have grown out of the existence of some selfish and self-important high-brows posing mathematical omniscience, who remind one of the man in Hiuen Tsang's story who wandered about staff in hand with a proud mien and lofty gait and a lighted torch on his head to show his pitiful concern to illuminate the darkness in which the ignorant people round him moved and had their being. But it is delightful to meet such great souls as Ramaswami Aiyar who give the lie direct to the adage, quite sober with all their mathematics and with whom you could engage for hours together in hearty mathematical conversation.

Any little original contribution of a junior was sure to evoke the keenest and warmest appreciation of Ramaswami Aiyar and

this is the best way for a senior to foster the true research spirit in the young aspiring soul. To quote Ramaswami Aiyar's own words, ill luck indeed led him away from a field, in which, had he continued to labour, he might have been of better service and perhaps put up solid work, nay, even a mathematical seminary would have sprung up after his own heart. He was often telling me that he carried a delightful recollection of his days in Mysore where he was a lecturer in Maharajah's College. If only he had continued in this College, what glorious marches would the college have stolen over the rival eighty-seven miles away!

FROM PROF. M. BHIMASENA RAO (RETD.), CENTRAL COLLEGE, BANGALORE

It was on the 11th of April 1924 that I first had a letter from the departed 'Professor'. From that day, for a period of nearly three years, a regular correspondence by almost every alternate post continued between us. Till 1935, we were in touch with each other at least off and on. It was during the three years 1924—'26 that most of our work on the Pedal and Contact Circle Theories was completed. I shall give here a few thoughts of his, culled out from the long series of letters that passed between us.

That he was a genuine lover of Mathematics was clear on every page that he wrote. Even though, out of modesty, he used to call himself 'a spent-out man', he was always able to find time for some Geometrical or analytical work every day in the midst of his taxing duties as a Deputy Collector. The desire he evinced to know more about my theories of Contact-circles and "get into grips with it" the eagerness with which he read Coolidge's Book on "Circles and Spheres" the interest he took in Lachlan's "Systems of Circles and Spheres" and the way he worked out many results pertaining to our special work, all reveal an absorbing interest in Mathematical study and Research, sure to kindle a like ambition in anyone coming into contact with him.

The next outstanding quality that strikes one is his unstinting and very generous appreciation of every work of merit that others might have accomplished. His admiration during his younger days of Mr. C. E. Youngman's Geometrical talents, his reference to my geometrical proof of the result that 'The Pedal circle cuts the nine-points circle at the sum of the angles at which it cuts the sides' as "one of the finest feats of Geometry" are a few among many such instances.

Combined with this generosity went the quality of modesty. One striking instance is the following:—"I have been a Prompter and a witness......and I suggest that the paper (On Miquel-Points and Circles and Centre-circles of a system of lines) may go in your own name".

Coming more closely to the main subject-matter of our correspondence, it was, while reading my articles on the Contact Circle, he gave the idea that, "for every confocal of an inscribed conic, a Contact circle may be defined in such a way that it becomes the Contact circle (as defined by me) when the Confocal becomes the thin ellipse PP′". In his letter of the 30th July 1924, he first explicitly states his desire to find out a clear connection between Pedal Circles and Contact Circles. This idea is pursued and he characteristically tries to find time during a six weeks' inspection tour of sub-treasuries to continue work on this subject and the extension of the Pedal Circle and orthopolar theories to the case of the Spherical triangle which, he says, "was an old ambition of mine".

In his letter of the 7th September 1924, he first gives his original and very fruitful idea (that has been the basis of a substantial amount of our joint work) as follows:—"Let ABC be an ordinary Spherical Triangle. Take the Complete Circles forming the sides. They will intersect in the Antipodal points A′, B′, C′. Rub off the arcs B′C′, C′A′, A′B′. Then we have a Spherical triangle A′B′C′ whose sides are $2\pi - a$, $2\pi - b$, $2\pi - c$, where a, b, c are the sides of the original triangle. Any point (l, m, n) connected with this triangle will be the same as the point (l, m, n) connected with the original triangle. The in- and the ex-circles and the Hart's circle are all the same," and adds "Extending the Conception of a Spherical triangle to include one whose sides are greater than π, the Contact Circle Theories for the triangle ABC becomes the Pedal Circle Theory for the triangle A′B′C′ and *vice versa*", a nice correction, which can be visualised on the Sphere.

There are several interesting results worked out by us which have not yet been published. I hope to be able to publish them in our journal at some future date.

Before concluding, I ought to mention the great kindness and hospitality he extended to me when I visited him for a fortnight at Chittoor.

GEOMETRY OF THE TRICUSPED HYPOCYCLOID

BY

A. A. KRISHNASWAMI AYYANGAR, *Maharajah's College, Mysore*

§ 1. It is perhaps in keeping with the spirit of a memorial volume that it should contain theses on subjects favourite to the departed soul concerned. I had the honour of collaborating with the late V. Ramaswami Aiyar about a decade ago on the topics developed in this paper and some of our joint results were published as theorems (without proof) in an abstract in the Proceedings of the Indian Science Congress, 1926, while a few have appeared as questions in the *Journal of the Indian Mathematical Society.* As no connected account of these and other results has yet been published †, I venture to take this opportunity to set forth, in some detail, the order of ideas which led me to work out the geometry of the tricusp in a simple and fruitful manner, stimulated now and then by the theorems communicated to me by the master.

§ 2. A Transversal Property

The properties of the tricusped hypocycloid can be studied from several points of view, one of the most fruitful of them being P. A. Mac Mahon's generalisation †† of the Simson-line envelope as that of the θ-pedal lines of a triangle. It is well known that the tangential equation of this envelope can be thrown into the form

$$\lambda^2 + \mu^2 = p\lambda^3 + q\lambda^2\mu + r\lambda\mu^2 + s\mu^3 \tag{1}$$

where (λ, μ) are the co-ordinates of the line whose cartesian point-equation is $\lambda x + \mu y + 1 = 0$. From this, it is evident that four tangents determine a unique tricusp, and therefore, from Mac Mahon's theorem, all tangents to a tricusp ††† are θ-pedals w.r.t. any tangent-triangle of the curve for a suitable θ.

† Before my collaboration with V. R. Aiyar began, I had contributed a paper entitled 'An Elementary Exposition of the allied properties of the Simson line, the Steiner envelope and the Orthopole' to the Third Conference of the Indian Mathematical Society held in 1921 (*J. I. M. S.* Vol. XIII No. 3. 1921, p. 85). This paper was published in the *Educational Review*, Madras, 1924 and a copy of it sent to V. R. Aiyar started our friendship and collaboration.

†† *Messenger of Mathematics* Vol. XII pp. 151-153 (1882).

††† I prefer to use the abbreviation 'tricusp' for the tricusped hypocycloid which touches the line at infinity at the circular points. The properties of the general tricusp need not be separately investigated as they are readily derived by projection. For some properties of the general tricusp, vide Question 1600, *J. I. M. S.*, Oct. 1931.

The pivotal theorem of this paper is

THEOREM I. *If any transversal cut the sides BC, CA, AB of a triangle at D, E, F such that*

$$BC \cdot BD + CA \cdot CE + AB \cdot AF = \Omega \text{ (a constant)}$$

(the segments being taken positively in the direction which makes the area of the triangle positive), then the transversal envelopes a fixed tricusp inscribed in the triangle ABC. †

Conversely, if ABC be any triangle circumscribed to a tricusp, and any tangent to the tricusp meet BC, CA, AB in D, E, F, then $BC \cdot BD + CA \cdot CE + AB \cdot AF$ *is constant.*

Proof: The circum-circles of the triangles AEF, BFD, CDE are known to have a common point. If P be this point and

$$\pi - P\hat{D}C = P\hat{E}A = \pi - P\hat{F}B = \theta, \text{ and if } D', E', F'$$

the projections of P on BC, CA, AB, then it is easily seen that

$$BC \cdot BD' + CA \cdot CE' + AB \cdot AF' = \tfrac{1}{2}\sum a^2 \text{ or } 2\triangle \sum \cot A \qquad (2)$$

$$\text{Also, } BC \cdot DD' + CA \cdot EE' + AB \cdot FF' = 2\triangle \cot\theta \qquad (3)$$

$$\therefore \quad \Omega = 2\triangle\left(\sum \cot A - \cot\theta\right) \qquad (4)$$

Hence, the transversal DE is a θ-pedal of ABC, where θ is given by (4), and its envelope is a tricusp.

Again, if $BD = (\Omega - c^2)/a$, $CE = 0$, $AF = c$, the condition of the theorem viz. $BC \cdot BD + CA \cdot CE + AB \cdot AF = \Omega$. (5)
is satisfied and BC is a tangent to the tricusp at D.

Similarly CA, AB are also tangents to the tricusp, which is thus inscribed in the triangle ABC.

The converse is easily seen to be true from the fact that four tangents determine a *unique* tricusp.

N.B.—The triangle ABC may be called the θ-tangent triangle of the tricusp, and θ the *pedal angle* of the triangle associated with the tricusp.

Cor 1. If $\Omega = 0$, $\theta =$ the Brocard angle of the triangle;
if $\Omega = (a^2 + b^2 + c^2)/2$, $\theta = \pi/2$.

Cor. 2. If $BD = (\Omega - b^2)/a$, $CE = b$, $AF = 0$, then AD satisfies (5) and is therefore another tangent from A to the tricusp; being a θ-pedal,

† Question 1458, *J. I. M. S.*, Dec. 1926.

AD makes an angle θ with BC. It is clear that no other tangent is possible through A and the tricusp is therefore a *class-cubic*.

Again, since no two θ-pedals can be parallel, only one tangent can be drawn to a tricusp from *any* point at infinity. This shows that the other two tangents must coincide with the line at infinity, which is therefore a bi-tangent.

Cor. 3. If D, E, F be the points of contact of BC, CA, AB with the tricusp, $BD=(\Omega-c^2)/a$, $CE=(\Omega-a^2)/b$, $AF=(\Omega-b^2)/c$ (6)

and so, $AB \cdot AF + AC \cdot AE = BC^2$ (7)

and two similar results, due to V. R. Aiyar. †

Cor. 4. If P bisects BC, D is the point of contact of BC with the tricusp, and X is the point where the third tangent from A meets BC, then

$DX/2=(c^2-b^2)/2a=$ the projection of PA on BC (8)

Thus the length DX is an *invariant* for all tricusps inscribed in the triangle ABC.

THEOREM II. *If ABC be a triangle circumscribing a tricusp, the third tangent through A and the isotomic conjugate in the angle A of the straight line joining A to the point of contact of BC with the tricusp are equally inclined to BC, the inclination being the pedal angle of the triangle associated with the tricusp.*

Proof: In the notation of Theorem I, cor. 4,

the projection of DA on $BC=DP+\frac{1}{2}DX=\frac{1}{2}(DD^1+DX)$ where D^1 is the symmetrique of D in P.

Hence, AX, AD^1 are equally inclined to BC and $C\hat{X}A=$ the pedal angle of the triangle ABC.

Cor. 1. In particular, when the pedal angle is $\frac{1}{2}\pi$, the third tangent through A coincides with the isotomic conjugate of AD, and conversely.

Also, if D coincides with B, $A\hat{C}B=$ the pedal angle.

Cor. 2. If AB is a tangent-chord of a tricusp and the tangents at A and B meet in C, then the pedal angle of the triangle ABC is the same as ACB and a right-angle, by cor. 1 above.

THEOREM III. *The pedal angle of a triangle associated with an inscribed tricusp is $\pi/2$, when, either* (α) *the third tangents through the*

† Question 1376, *J. I. M. S.* Feb. 1925. It is this little beautiful result (7) that stimulated me to discover the pivotal theorem of this paper.

vertices are concurrent or (β) *the straight lines joining the vertices to the points of contact of the opposite sides are concurrent; and conversely.*

This result follows readily from elementary geometry since the only concurrent lines from the vertices which make equal angles with the opposite sides of a triangle are the altitudes of the triangle.

Cor: (α) and (β) are equivalent.

§ 2·1. **A Special set of line-co-ordinates.**

If $BD = l_a$, $CE = l_b$, $AF = l_c$, (l_a, l_b, l_c) may be designated as a *special set of line-co-ordinates* of the line DE, when ABC is the triangle of reference.

The equation of an inscribed tricusp, in these co-ordinates, is

$$al_a + bl_b + cl_c = \Omega \tag{9}$$

$$\text{or} \qquad l_a \sin A + l_b \sin B + l_c \sin C = \Omega^1 \tag{10}$$

If $(l_p{}^i, l_q{}^i, l_r{}^i)$, $(l_p{}^j, l_q{}^j, l_r{}^j)$ be the *special line-co-ordinates* of two tangents t_i, t to a tricusp, when referred to the triangle formed by three other tangents t_p, t_q, t_r, then, by virtue of (10),

$$\sum (l_p{}^i - l_p{}^j) \sin\theta_{qr} = 0 \tag{11}$$

where θ_{qr} is the angle between the tangents t_q, t_r.

An elegant application of this result is

THEOREM IV. (*Aiyar's Theorem*) †: *If five straight lines be tangents to a three-cusped hypocycloid, the foci of the five parabolæ touching the lines taken four at a time are collinear.*

Proof: Let the five tangents be t_1, t_2, t_3, t_4, t_5 and F_i the focus of the parabola touching the four tangents other than t_i $(i = 1, 2, 3, 4, 5)$

If θ be the pedal angle associated with the tricusp for the tangent-triangle whose sides are t_3, t_4, t_5, then F_1, F_2 are the poles of t_2, t_1 regarded as θ-pedals for this triangle.

$$\text{Hence } F_1F_2 = (l_3{}^4 - l_3{}^5) \sin\theta_{12} / \sin\theta_{45}$$

$$\text{Similarly, } F_2F_3 = (l_1{}^4 - l_1{}^5) \sin\theta_{23} / \sin\theta_{45}$$

$$\text{and } F_3F_1 = (l_2{}^4 - l_2{}^5) \sin\theta_{31} / \sin\theta_{45}$$

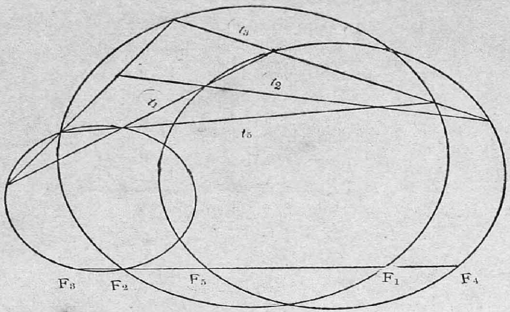
where $(l_1{}^4, l_2{}^4, l_3{}^4)$ and $(l_1{}^5, l_2{}^5, l_3{}^5)$ are the *special line-co-ordinates* of t_4, t_5 referred to the triangle formed by t_1, t_2, t_3.

$$\therefore \quad F_1F_2 + F_2F_3 + F_3F_1 = \operatorname{cosec}\theta_{45} \sum \sin\theta_{12} (l_3{}^4 - l_3{}^5)$$

$$= 0 \qquad \text{by (11)}$$

and F_1, F_2, F_3 are collinear.

† Question 1250, *J. I. M. S.* August 1922.



Thus, any three and therefore all the five foci are collinear.

Cor: If t_1, t_2 t_3 t_4 t_5 be any five tangents to a tricusp forming a Miquel pentagon, the corresponding Miquel circle opens out into a straight line. †

§ 2·2. **Mean-Centres.**

Definition: Given n tangents, viz. t_1, t_2, $\cdot\cdot$ t_n, to a tricusp, the mean centre, say M_t, of their intersections with any tangent t to the tricusp for given multiples $(\lambda_1, \lambda_2, \cdot\cdot\lambda_n)$ where $\sum\lambda_i\neq 0$, is defined as a *mean-centre* associated with the n tangents, and M_t is said to appertain to the tangent t.

THEOREM V. *If three mean-centres associated with* n *given tangents to a tricusp are collinear, then all the mean-centres are collinear; and the line of collinearity is a tangent to the tricusp at the mean-centre appertaining to it.*

Proof: Take for the triangle of reference any tangent triangle ABC of the tricusp, with Ω as the associated constant. Denote the mean-centres appertaining to the sides by M_a, M_b, M_c and the intersections of a tangent t_i with the sides by D_i, E_i, F_i and Theorem I gives

$$BC\cdot BD_i + CA\cdot CE_i + AB\cdot AF_i = \Omega$$

$$\therefore \quad BC\sum\lambda_i BD_i + CA\sum\lambda_i CE_i + AB\sum\lambda_i AF_i = \Omega\sum\lambda_i$$

Hence $BC\cdot BM_a + CA\cdot CM_b + AB\cdot AM_c = \Omega$ (12)

an important result relating to three mean-centres.

† Cf. Pp. 151, 152, Casey's *Sequel to Euclid*, Longmans, 1886; Pp. 89, 90, J. L. Coolidge, *A Treatise on the Circle and the Sphere*, Oxford, 1916.

Now, if M_a, M_b, M_c be collinear, Theorem I makes it evident that the line of collinearity is a tangent to the tricusp. Let M be the mean-centre appertaining to this tangent and P its point of contact.

Considering AM_bM_c as the triangle of reference, and using the property of three mean-centres,

$$M_cM_b \cdot M_cP + AM_c^2 = M_cM_b \cdot M_cM + AM_c^2$$

$\therefore$ M coincides with P.

Again, if any other tangent to the tricusp not passing through M_c meet AB, M_bM_c in U, V, and M′ the mean-centre appertaining to it, consider M_cUV as the triangle of reference and apply (5) and (12), so that $M_cP \cdot M_cV + UM_c^2 = M_cP \cdot M_cV + VM' \cdot VU + UM_c^2$

$\therefore$ M′ coincides with V.

To prove that M_c appertains also to another tangent through it besides AB, consider as the triangle of reference the one formed by this tangent, M_bM_c and BC and apply (5) and (12).

Similarly, M_b appertains also to two tangents and so every point on M_bM_c appertains to two tangents.

Thus, all the mean-centres lie on the line which passes through M_a, M_b, M_c.

Cor: When three mean-centres are not collinear, no other three can be collinear and hence the locus of the mean-centres is a conic. In this case, two different tangents cannot have the same mean-centre.

§ 3. **Concurrent Tangent Triads.**

In Theorem I cor. 2, it was proved that from any point three and only three tangent lines can be drawn to a tricusp and these may be considered as forming an infinitesimal triangle to which the result (10) can apply.†

THEOREM VI. *If P_1, P_2, P_3 be the points of intersection of any tangent to a tricusp with three concurrent tangents OT_1, OT_2, OT_3 whose points of contact are T_1, T_2, T_3 then*

$$\sum OP_1/OT_1 = 1 \text{ and } \sum 1/OP_1 \cdot OT_1 = 0$$

† I owe this suggestion to V. R. Aiyar,

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Since $\sum OP_1 \sin T_2OT_3$ is constant, it may be equated to the particular values obtained by considering OT_1, OT_2, OT_3 as the transversal tangents, so that

$$OT_1 \sin T_2OT_3 = OT_2 \sin T_3OT_1 = OT_3 \sin T_1OT_2$$
$$= \sum OP_1 \sin T_2OT_3 \qquad (13)$$

$$\sum OP_1/OT_1 = \frac{\sum OP_1 \sin T_2OT_3}{OT_1 \sin T_2OT_3} = 1 \qquad (14)$$

Again $\triangle OP_2P_3 + \triangle OP_3P_1 + \triangle OP_1P_2 = 0$, leads immediately to

$$\sum OP_2 \cdot OP_3/OT_1 = 0$$

$$\therefore \quad \sum 1/OP_1 \cdot OT_1 = 0 \qquad (15)$$

Cor. 1. If p_1, p_2, p_3, p be the perpendiculars from T_1, T_2, T_3, O on any tangent to the tricusp, then

$$\frac{1}{p} = \frac{1}{p-p_1} + \frac{1}{p-p_2} + \frac{1}{p-p_3} \qquad (16)$$

Cor. 2. If OT_1, OT_2, OT_3 make equal angles with each other, then $OT_1 = OT_2 = OT_3$ and $\sum OP_1 = OT_1$, $T_1T_2T_3$ becoming equilateral.

Cor. 3. If OT_3 is produced to cut the circle OT_1T_2 in T, then $OT = 2OT_3$.

THEOREM VII. *If the tangents at T_1, T_2, T_3 to a tricusp meet in O, the isogonal conjugate of OT_3 in the angle T_1OT_2 bisects T_1T_2 and is parallel to the other tangent from T_3.*

Proof: Let the isogonal conjugate of OT_3 meet T_1T_2 in K.

The perpendicular from T_1 on OK

$$= OT_1 \sin T_1OK$$
$$= OT_1 \sin T_2OT_3$$
$$= OT_2 \sin T_3OT_1$$
$$= OT_2 \sin T_2OK$$

= the perpendicular from T_2 on OK.

$\therefore$ K is the middle point of T_1T_2.

If the other tangent from T_3 meet OT_1 in L and OT_2 (produced, if necessary) meet T_3L in V, then, by Theorem I cor. 2,

$$O\hat{V}T_3 = T_3\hat{O}T_1 = V\hat{O}K$$

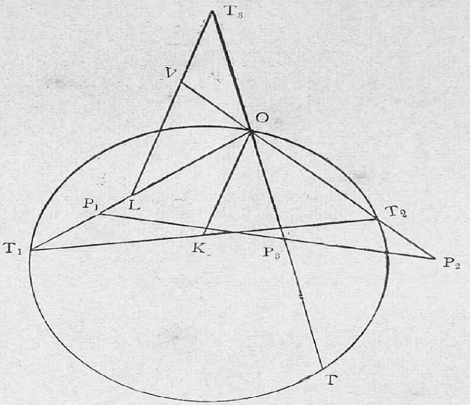
which shows that T_3L is parallel to OK.

Cor. 1. If $OT_1 = OT_2$, OT_3 itself passes through the middle point of T_1T_2 and the third tangent from T_3 coincides with OT_3 thereby indicating that T_3 is a cusp.

Hence, if $OT_1 = OT_2 = OT_3$, T_1, T_2, T_3 are all cusps and OT_1, OT_2, OT_3 are cuspidal tangents.

Cor. 2. The tangents from T_1, T_2, T_3 other than OT_1, OT_2, OT_3 are also concurrent and the line joining O to the point of concurrence is divided in the ratio 1 : 2 at the centroid of $T_1T_2T_3$.

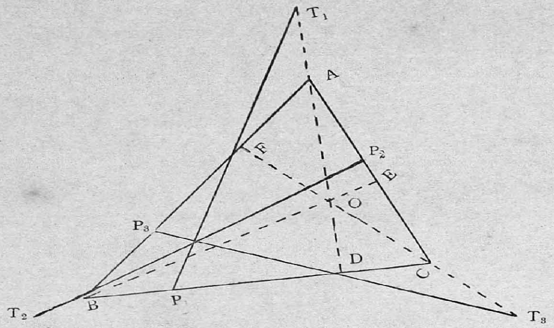
Cor. 3. † O is a focus of the maximum inscribed ellipse of the triangle $T_1T_2T_3$ and since there are two real foci for this ellipse, two tricusps can in general be drawn such that the tangents at T_1, T_2, T_3 may be concurrent. If the triangle $T_1T_2T_3$ be equilateral, then there is only one tricusp having this property and it has cusps at T_1, T_2, T_3



THEOREM VIII. *If ABC is π/2-tangent triangle to a tricusp and T_1, T_2, T_3 the points of contact of the third tangents from A, B, C and P_1, P_2, P_3 the points of contact of BC, CA, AB with the tricusp, then T_1P_1, T_2P_2, T_3P_3 are concurrent at a point which is the symmetrique of the orthocentre of the pedal triangle of ABC w. r. t. its circumcentre.*

Proof: If D, E, F be the feet of the altitudes of the triangle ABC with orthocentre O, the isotomic conjugate property of Theorem II shows that $BP_1 = DC$ and $AT_1 = OD$.

† This result is due to V. Ramaswami Aiyar.



$\therefore$ P_1T_1 is parallel to the line joining the middle points of AO and BC and therefore perpendicular to EF.

Hence P_1T_1 is the reflexion of the altitude from D to EF in the circumcentre of the triangle DEF and passes through the symmetrique of the orthocentre of DEF in its circumcentre.

P_2T_2, P_3T_3 also pass through the same point.

Cor. 1. P_1T_1 is also a tangent to the tricusp since it is parallel to the isogonal conjugate of AD in the angle BAC.

So are P_2T_2 and P_3T_3.

Cor. 2. The straight lines through A, B, C parallel to P_1T_1, P_2T_2, P_3T_3 respectively meet in the circumcentre of ABC and therefore by the involution property of a quadrangle the point-pairs at ∞ on (BC, P_1T_1), (CA, P_2T_2) and (AB, P_3T_3) belong to the same involution.

THEOREM IX. *Tangent pairs from points on any given tangent to a tricusp meet the line at infinity in point-pairs of the same involution, of which the circular points are members. The double points of the involution are the points at infinity on the tangents at the extremities of the tangent-chord.*

Proof: Take any point P_1 at infinity and let the unique tangent from P_1 to the tricusp meet the given tangent at T_1. The third tangent from T_1 meets the line at infinity at Q_1. Repeating the construction from Q_1, P_1 is obtained and so, (P_1, Q_1) are in involutory correspondence to each other and therefore belong to an involution.

If P_1, Q_1 coincide, they coincide with a double point of the involution and since in this case the tangents from P_1, Q_1 also coincide, the coincident tangent-pair is the tangent at a point where the given tangent meets the tricusp. It has been shown that the tangents at the extremities of a tangent-chord cut at right-angles (Theorem II Cor. 2.) and therefore their points at ∞ which are the double points separate the circular points harmonically. Hence the circular points also belong to the involution set up by the points at infinity on the tangent pairs.

N.B.—This involution is called the involution at infinity corresponding to the given tangent and is different for different tangents.

Cor 1. If T be the point at infinity on a given tangent t and T′ the point at infinity on the other tangent through the point of contact of t, (T, T′) belong to the involution corresponding to t.

Cor. 2. The tricusp is of order four, since no tangent to it can meet it in more than four points.

Cor. 3. The locus of the intersections of the θ-pedal lines parallel to the line-pairs of an involution pencil of which the double-lines are orthogonal lines, is another θ-pedal.

§ 4. Ellipses having triple contact with a tricusp.

THEOREM X. *The locus of the intersections of tangents to a tricusp from point-pairs of an involution at infinity, say J, which does not include the pair of circular points as members, is an ellipse having triple contact with the tricusp.*

Proof: Let t be any tangent to the tricusp whose point at infinity is T. If T′ be the mate of T in the involution J, tangents from T, T′ intersect on t and the required locus, say S. Again J and the involution at infinity corresponding to t have one pair of common points, say K, K′ and the intersection of tangents from K, K′ is another common point of t and S and there are no more.

Hence S is a conic.

The mates of the circular points at infinity in the involution J are the points at infinity on S. These being imaginary, S is an ellipse.

The tangent 't' to the tricusp will touch this ellipse when the pairs (T, T′), (K, K′) coincide and in this case the tangent from T′ meets t at its point of contact with the tricusp (Theorem IX, Cor. 1).

This shows that every common tangent to the ellipse and the tricusp touches both at the same point.

Therefore the six common tangents of the ellipse and the tricusp reduce to three which touch both the curves at the same point.

The triple contact property is thus established.

Cor. 1. If the involution at infinity has the circular points as double points, the corresponding locus is the inscribed circle of the tricusp. Since the tangents from point-pairs of the involution must be at right angles, the in-circle is also the locus of the intersection of perpendicular tangents—a well-known result.

Cor. 2. The locus of the intersections of θ-pedals of the extremities of chords through a given point is a conic having triple contact with the pedal envelope.†

THEOREM XI. *Every conic which has triple contact with a tricusp is inscribed in a $\pi/2$-tangent triangle and corresponds to an involution at infinity.*

Proof: The first part of the theorem follows immediately from Theorem III and the well-known property of conics, that the lines joining the vertices of a triangle to the points of contact with the opposite sides of any inscribed conic are concurrent.

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† When $\theta=\frac{\pi}{2}$, this is the well-known orthopolar conic first discovered by J. Neuberg in 1910 (*Bulletin de l'Academie royale de Belgique*, juillet-aout, 1910) and the tritangent property of this conic was studied by R. Goormaghtigh in *Nouvelles Annales* 1913, where he refers to an earlier writer J. Lemaire on the same topic. O. J. Ramler has studied the properties of this ellipse in the *American Mathematical Monthly*, March 1930, not aware of similar work by other writers. This induced R. Goormaghtigh to come out with his notes 'On Some loci connected with Orthopole-Geometry' in the same journal, August-September 1930. He remarks thus on O. J. Ramler's article: 'Several properties obtained in that article have already been mentioned and it may be interesting to recall, with some notes on the literature of the subject, a few other theorems about the loci considered. . . . .

The fact that the conic, locus of the orthopoles, is always an ellipse is proved by A. A. Krishnaswami Ayyangar in his paper 'A simple exposition of the allied properties of the Simson-line, the Steiner envelope, and the orthopole' (*Educational Review*, Madras, October 1924)...The properties about the lengths of the axes of the orthopolar conic are also to be found in Ayyangar's paper; it contains the following determination of the directions of the axes: they are perpendicular to the Simson-lines of the extremities of the circum-diameter passing through P. . . . .'

I may add a few more references; Question 775, P. 127, Wolstenholme's *Mathematical Problems*, 1878; and V. R. Aiyar's Question 1126, *J. I. M. S.* October 1920 and questions due to other writers in *J. I. M. S.* viz., Questions 238, 321, 328, and 270.

This merely illustrates the historical fact that an interesting result is never allowed to be forgotten by the world and re-discoveries occur time and again.

By Theorem VIII Cor. 2, the points at infinity on the tangent pairs to the tricusp at its points of contact with the $\pi/2$-tangent triangle belong to the same involution; and Theorem X shows that this involution corresponds to an ellipse which has triple contact with the tricusp at the same points as the conic in question and therefore the two conics coincide.

THEOREM XII. (*Aiyar's Theorem*). *The locus of the mean-centres associated with* **n** *given tangents to a tricusp is, in general, an ellipse having triple contact with the tricusp.*

Proof: Take any point P_1 at infinity and t_1 the tangent from it to the tricusp. If M_1 be the mean-centre appertaining to t_1, it is possible, in general, to find only one other mean-centre on it, say M which appertains to another tangent t. This is by virtue of Theorem V. Cor.

Through M, there lies a third tangent t_2 whose point at infinity is, say, P_2 and mean-centre is M_2.

The above process started from P_2 will lead to P_1 and thus the points (P_1, P_2) are in involutory correspondence, which shows that the locus of M is a tritangent ellipse.

N.B.—A particular case of this theorem is dealt with in detail by R. Goormaghtigh in *Nouvelles Annales*, *t.* XV, October 1915; and Question 1462 of V. R. Aiyar, *J. I. M. S.* February 1927 contains Goormaghtigh's theorem and the above generalisation.

THEOREM XIII. *If the straight line joining two mean-centres passes through a given point, the locus of the intersection of the tangents to which these mean-centres appertain is, in general, a tritangent ellipse.*

As in the proof of the previous theorems, it can be shown that an involution at infinity is generated by the points at infinity on the tangent-pairs to which appertain two mean-centres whose join passes through the given point; hence the theorem follows.

§ 5. Hexad of common tangents to a conic and a tricusp.

It is proposed to prove in this section that the hexad of common tangents to any conic and a tricusp, if split up in any manner into two triads, the pedal angles of the tangent-triangles thus obtained are always supplementary. The pedal angles will become equal only when each is a right-angle. This explains why when the conic has triple contact with the tricusp, the pedal angle of the triangle formed by the tangents at the points of contact is a right angle (Theorem XI *supra.*)

Two elementary properties of θ-pedals may be referred to in this connection, without proof.

THEOREM XIV. *If $A_1 A_2 A_3$, $P_1 P_2 P_3$ be two triangles inscribed in a circle such that the θ-pedals of P_1, P_2, P_3 w.r.t. $A_1 A_2 A_3$ are concurrent, then the θ-pedal of P_i makes with $P_j P_k$ an angle equal to π-θ $(i, j, k \equiv$ a permutation of $1, 2, 3)$*†

THEOREM XV. *If $A_1 A_2 A_3$, $B_1 B_2 B_3$ be two triangles such that $A_i A_j$ is a θ-pedal w.r.t. $B_1 B_2 B_3$ and $B_i B_j$ a $(\pi$-$\theta)$-pedal w.r.t. $A_1 A_2 A_3$, then the poles of the sides of either triangle considered as oblique pedals w.r.t. the other form a triangle homothetic to the first triangle.*

This theorem can be easily inferred from the preceding, since the third pedal through B_k makes the same angle with $B_i B_j$ and the line joining the poles of $B_i B_k$ and $B_j B_k$ w.r.t. $A_1 A_2 A_3$.

THEOREM XVI. *If A_1, A_2, A_3, P_1, P_2 be any five points on a circle, then the sides of the triangle $A_1 A_2 A_3$, P_1P_2 and the θ-pedals of P_1, P_2 w.r.t. it, all touch a conic.*

Proof: Draw the chord $P_2 P_3$ parallel to the θ-pedal of P_1. Then $P_1 P_3$, $P_1 P_2$ are parallel to the θ-pedals of P_2, P_3 respectively.

Let A_1L be a chord parallel to P_2P_3 and P_1L meet A_2A_3 in M. If UV be a chord through M parallel to P_2P_3, it is evidently the θ-pedal of P_1.

For the range of conics touching $A_1 A_2$, $A_2 A_3$, $A_3 A_1$ and UV, the tangent-pairs from P_1 form an involution, say J, to which belong (i) P_1A_1, P_1L, and (ii) P_1U, P_1V since P_1UV and $A_1A_2A_3$ being inscribed in a circle must circumscribe a conic.

A_1L, UV, P_2P_3 being parallel chords, the lines joining their extremities to P_1 form an involution, which is identical with J having two pairs of lines in common. Hence P_1P_2, P_1P_3 also touch a conic of the range, which can be none other than the common inscribed conic, say Γ, of the triangles $A_1A_2A_3$, $P_1P_2P_3$.

Thus the θ-pedal of P_1 touches Γ and similarly the θ-pedals of P_2, P_3 touch the same conic.

Cor: The θ-pedals of P_1, P_2, P_3 being parallel to the sides of the triangle $P_1P_2P_3$ form a triangle homothetic to and congruent with $P_1P_2P_3$.

† Goormaghtigh: The Orthopole, pp. 117, 118, The *Tohoku Mathematical Journal*, July 1926.

The centre of Γ is midway between the circum-centres of $A_1A_2A_3$ and the triangle formed by the θ-pedals.

† THEOREM XVII. *Two tangent-triangles to a tricusp whose pedal angles are supplementary circumscribe a conic whose centre is midway between the circumcentres of the two triangles.*

This follows immediately from the two preceding theorems.

THEOREM XVIII. *The necessary and sufficient condition that two tangent-triangles to a tricusp may be circumscribed about the same conic is that their pedal angles should be supplementary.*

The preceding theorem has established the sufficiency of the condition.

Assuming θ, ϕ to be the pedal angles of two tangent-triangles say $A_1A_2A_3$, $B_1B_2B_3$ to a tricusp, whose sides touch a conic say S, draw a tangent t to the tricusp which makes an angle $(\pi-\theta)$ with the third tangent from a vertex say B_1 of the ϕ-tangent-triangle. By the preceding theorem since B_1B_2, B_1B_3 and t form a $(\pi-\theta)$-tangent triangle, t is a tangent to the conic which touches the sides of the θ- and $(\pi-\theta)$-tangent triangles and therefore t touches S.

Now, t must coincide with B_2B_3, for otherwise there will be *seven* common tangents to S and the tricusp.

This proves the theorem.

Cor: The necessary and sufficient condition for two tangent triangles to a tricusp to be inscribed in a conic, is that their pedal angles should be supplementary.

Note: It is easily seen that, within *any* given conic, we can inscribe two and only two tangent-triangles to a tricusp, since more than two implies equality of the pedal angles, which are therefore right-angles. It is interesting to notice that when the pedal angles of two tangent triangles are each $\pi/2$, we can inscribe an infinite number of such tangent-triangles in the common circumconic of the two triangles, which is a rectangular hyperbola whose asymptotes touch the tricusp.

† This is a generalisation of Question 1431 of V. R. Aiyar, *J. I. M. S.* June 1926. Mr. T. R. Raghava Sastri has given a wrong generalisation in Question 1629, *J. I. M. S.* October 1932.

AT CLOSE RANGE

BY

S. R. RANGANATHAN, *Madras.*

It was a cold December morning. It was 29th December 1916. About seventy savants coming from all the corners of the Indian continent had assembled at the First Conference convened by the Indian Mathematical Society, which was just completing its first decade of existence. On that day, we met in the junior English lecture hall of the Presidency College, Madras.

A tall, grave, grey-haired gentleman in white long coat was in the chair. They said he was the famous Professor B. Hanumantha Row and that he was the first President of the Indian Mathematical Society. Professor A. C. L. Wilkinson, the President of the year, was announced and he ascended the platform and gave a summary of the properties of the Nine point circle, with the aid of his *Fundamental triangle.* While this paper was going on, Professor P. V. Seshu Ayyar, who was the only one among the leaders of the Conference whom I knew and who, as the local Secretary, was in charge of the affairs, thrust into my hands my little paper entitled *A review of Indian Mathematics* and said I would be called upon as the fourth. This announcement made me forget where I was and I could not hear the two succeeding papers. The Chair called my name. I was on the platform—the first time I faced an audience. I was still a student—having four more months to complete my term. I did not know what I was doing. Professor Hanumantha Row pulled me by his left hand and whispered into my ears "Don't be afraid. It is good. Be a little louder." In another ten minutes, I was discharged, and a Parsee gentleman (Professor Sanjana) was on his legs, saying something about my paper. But, I leapt out of the platform, as the audience was clapping hands. I was perspiring profusely in that cold morning. I went into the adjoining southern verandah to hide myself.

First Contact

When I was near the corner, my right hand was in the grip of a very strong massive hand. My fingers were aching. I could not bear that handshake. I still remember it. I was about to cry and I looked up. I saw a pair of eyes which were sparkling and bewitching. They were set on a broad face with a broad smile. A

giant-like stalwart figure was holding me by the hand. I did not know who he was, although I had seen that figure among the bigwigs of the conference.

"Are you related to Mr............? (I cannot recollect that name).

I said "No".

"I am very glad that my Society is able to bring to light young boys of such varied interests. I am proud of you and Vaidyanathaswamy............"

I was puzzled, as I did not know, who he was. There was nobody else in the corner, as every one else was in the hall listening to the next paper. Hence, I made bold to ask him who he was. "I am V. Ramaswami Ayyar" he said, "I started this Society ten years ago. My heart delights to find young boys like you take interest in Mathematics. Good! Good!!........"

At this stage, Professor Seshu Ayyar came and took away Ramaswamy Ayyar.

The Conference concluded at 11 A.M. I was just getting down the stairs opposite the Powell's Statue, in an absent-minded condition. Professor Ramaswamy Ayyar put his hands round me and told Professor Seshu Ayyar, "We shall take this young man with us."

Second Contact

It was about 8-30 A.M., January 9th, 1919. The Bombay Mail steamed into the Adoni Station. I was on the platform. A voice from one of the second class compartments called out my name. I turned round and saw the impressive figure of Ramaswamy Ayyar greeting me with extended hands, a loving smile and the pair of eyes that appeared to be themselves smiling. There were several other Madras mathematicians in that compartment, all bound to the Second Conference at Bombay. Professor Ramaswamy Ayyar asked me to get in. I said I was travelling third. He said "Does not matter. You will be my 'servant'! Get in." My luggage was deposited into a neighbouring third class compartment and I was dragged into his compartment by the mighty professor. The moment I entered, he began to talk about Bernoulli's Numbers and Bernoulli's Polynomials. As we approached the next stop, he asked me to go to my compartment and bring out whatever food I had brought. He said, "I have two boxes of food, all prepared days ago at Tinnevelly. I must taste your fresh food from Adoni." I brought it. He went on telling us

stories about Ramanujam and the genesis of the Indian Mathematical Society and before we reached Raichur, there was nothing left in my tiffin carrier. Throughout the journey, Mr. Ramaswamy Ayyar was the centre of all our attention, whether it was fun or serious mathematics.

Third Contact

In July 1921 Professor Seshu Ayyar received a letter from Professor Ramaswamy Ayyar, which said "This time I want Ranganathan to be my bodyguard. I learn that he has joined the staff of the Presidency College." He came to attend the meeting of the Board of Studies. I went to Egmore to receive him. As soon as the Boat Mail pulled up at the reception platform, the Professor stepped out with the same old smile and the same old sparkling, bewitching eyes and, in addition, a packet in his hands. He placed the packet into my hands saying, with all love and affection, "Here is something I want to transmit to you. These have helped me during the last few years to discharge my duties in the M. A. Board. It is not proper that I should be there any longer. I want you to take my place. These Tripos papers will be of help to you, when you become an examiner." In another year, I was an examiner, as he wished.

Next evening, we went to George Town, myself acting as his "bodyguard". The first thing he did was to get into a hotel in Broadway. He ordered some two rupees worth of dishes. I could not take more than four annas worth. But he finished the lot remarking "If you don't eat well, you cannot do mathematics." A little later, he said, "Let us get into a bus. I have not yet experienced a bus ride." Buses had just then been introduced into our city. When we got in at Broadway, the conductor refused to carry him unless he paid double fare. I was annoyed at this impertinence of the fellow, but the Professor paid him the double fare as demanded and silenced me by diverting my attention with the question "What subjects are you teaching?" When I said "Algebra", he jumped with enthusiasm and said, "I shall reveal to you tonight some of the secrets of Inequalities". I went to Professor Seshu Ayyar's house at 9 P.M. with a note-book and pencil. Ramaswamy Ayyar was waiting for me. He started from the very beginning and led me on to the most involved of the inequalities, I had ever come across—all tackled by the simplest and the most direct methods, involving nothing more advanced than the ordinary manipulations but handled with a marvellous dexterity and ease. It

was 3 A.M., when I came home with the note-book full of theorems and examples, all dictated by the Professor—nay, the Deputy Collector—without the aid of a scrap of paper.

At Close Range

From 1925, we came to know each other more intimately. He used to visit Madras regularly every year and he gave me the pleasure of his company on many occasions. All day and night, whether bathing, or eating or dressing, he used to discuss mathematics, mathematics and mathematics alone. His engrossment in the mathematical talks was so deep, that the children in the house used to stand looking at him with wonder and affection. We used to have long walks in the morning along the Marina. Any little incident would start him on some mathematical theorem. One day, it was the geometrical design, in which the plants in a square of the Marina were laid out. He said that those patterns and the patterns of *Kolam* were a source of stimulation to his geometrical investigations. That evening, he was to lecture at the Mathematical Association of the Presidency College on the "Orthopolar theory". But the train of thoughts that got started in his head at the sight of the pattern of plants that morning had got such a possession of him, that, in the evening lecture, he started with the various geometrical patterns that arose out of Euclid I. 47 and went deeper and deeper into the subject, all with the aid of very elementary geometrical methods and led the students to so many interesting results and generalisations to n-gons and n-space that flowed from I. 47, that the hour was far exceeded and the meeting had to close. All the students enjoyed the exposition with remarkable pleasure; but only they thought that it was all orthopolar theory and that, after all, the orthopolar theory was so simple!

Absolute Absorption

This self-forgetfulness, and deep absorption in his problems was characteristic of him. He could not be weaned away from his problem by anything, until he had gone to the end of it, and discovered extremely original modes of attack. He was also fond of "setting a thief to catch a thief". If any problem presented itself, he would first examine the chief degenerate or particular cases of the problem, which had an obvious solution and would first list them. He had the flair to pick out just those particular cases or degenerate cases that could be turned to good account to attack the problem in its generality. By repeated applications of these particular cases, he would, as if by magic, produce at the other end, a complete,

exhaustive, elegant and extremely simple solution of the general problem. The last and the most characteristic illustration of this method of his was found in—what turned out to be, alas, his swan song—in his University Lectures on the "Fermat Problem" in November 1935.

An ambition of his life

This last and only opportunity he had to expound in the University a piece of mathematics of his own was a source of immense satisfaction to him. For some years, his great ambition was to get himself associated officially with some University in the pursuit of research. He was frequently asking me whether there would be any possibility of his being recognised as an honorary teacher in the Madras or the Annamalai University. Hence his delight at opportunity he was given at last to deliver a course of special University Lectures. After they were over, when he came home and was relaxing himself I found him in high spirits, humming loudly, "I have fulfilled my life's desire". When I told him that he was in excellent form in his lectures, he was immensely pleased and the glow in his eyes redoubled.

Parting of the ways

This last visit of his showed him a different man in several respects. Quite unlike on earlier occasions, he was prepared—nay he was eager—to have some of the Sanskrit classics read and interpreted to him. He asked me to do a few outstanding chapters of the Ramayana and Bhagavata. I was interpreting the passages to him in mixed Tamil and English. When I looked up at him, I found him all in tears. He said that he was overpowered with the extraordinary human touches of the passages read. He expressed a great regret that he had not learnt Sanskrit while young. It was then that he gave a characteristic review of his life dividing it into octaves. In the first octave, he said, he was nearest to nature. The next eight years was the octave of formal school days, being led by others. Seventeenth to the twenty-fourth year was the creative octave when he developed his most distinctive features, mentally, physically and morally. The fourth octave was the octave of knocks of the reality of life. The fifth octave was the octave of achievement to which belonged the foundation of the Indian Mathematical Society, the publication of some of his papers and above all the part he played in the discovery of Ramanujan. The sixth octave was a period of disappointment and disillusionment. The seventh octave was that of marking time in anticipation of retirement in a mood of sullen

resignation and reconciliation. He said that he was just completing his eight octave which he had spent in retirement, review and rumination. This was also the period when he picked up several of the creative pieces of work left unfinished in the third octave. The generalised Fermat problem was one of the problems which had gone clean out of his mind in his twenty-fourth year and shot into his attention after a period of about 40 years. In the twenty-fourth year he could not solve the problem and it disappeared from his mind. But evidently, he said, that was all along shaping itself in his sub-conscious mind, and one fine morning just a few months before the time of speaking, the problem suddenly came to him along with several suggestions for its solution.

He said he was just entering into the ninth octave when he had a great yearning to pursue other things, things of the spirit and problems like that of the Soul, Life and God. He said that the episodes that were read to him from the Ramayana and the Ḅhagavata were having profound meanings for him. He hoped to pursue this new phase of his life in the next few years. Thus his mathematical and worldly ambitions had been nearly fulfilled and he definitely set himself towards a new life along spiritual lines. The call came when he was in this parting of the ways.

A pathetic reunion

Another event occurred during his last visit. Prof. Seshu Aiyar was down in the General Hospital with cancer. A week before Ramaswami Aiyar's arrival at Madras, I met Prof. Seshu Aiyar and told him that Ramaswami Aiyar was coming to deliver some University lectures. Seshu Aiyar was quite cheerful on that day and he asked me to tell Ramaswami Aiyar that he was too weak to go to the Senate House to attend his lectures and asked him to see him in the hospital after the lectures were over. A week later, Ramaswami Aiyar, G. A. Srinivasa Aiyar and myself went to the hospital. But within that week, Prof. Seshu Aiyar had sunk rapidly. His face had withered, his eyes had gone deep, coma had set in. He could speak only in a whisper. As soon as we approached his bed, Ramaswami Aiyar burst into tears and sobs. Holding Seshu Aiyar's hands, he sobbed out, "Seshu Aiyar, you should not die before me." The scene was too overpowering for Mr. G. A. Srinivasa Aiyar and myself. We went aside. After some time, in the interest of the patient, we interceded and with great difficulty took Ramaswami Aiyar away. This pathetic scene disclosed to us a most delicate trait of Ramaswami Aiyar which did not manifest itself to us of the

younger generation in such a pronounced form till then. On our way home, he was recollecting his earlier days when the Indian Mathematical Society was founded and he was also recounting how most of his co-workers of those days were rapidly disappearing. But he suddenly struck an optimistic note with glittering eyes and he began to recount all the younger men of the present generation who were making a mark in the mathematical field. He mentioned about half a dozen names, I need not state them here; and he said that he was quite confident that mathematical research in this part of the world was quite safe in their hands.

RAMASWAMI AIYAR'S NOTE-BOOKS

BY

A. NARASINGA RAO, *Annamalainagar*.

Ramaswami Aiyar has left behind him three quarto and about a dozen folio note-books.

The former, which form a record of his earlier work, comprise altogether about a thousand pages of written matter, subdivided into 456 paragraphs, each dealing with a particular result and its applications. Generally, each result is dated, the earliest entry bearing the date 4—8—1891 (when he was just 20 years old) and the record running almost continuously till the end of 1896 after which there are only 5 entries in 1897, four in 1900 and one in 1913. It is possible that by this time the first folio had come into existence as the last quarto is only half full.

The folios are a record of rough work and calculations, but when a result has been verified, it is formally enunciated and dated. These investigations are later than those in the quartos and run on till the 9th December 1935 a few weeks before his death.

These note-books are something more than a dry record of discoveries. They pulsate with the hopes and the apprehensions of the young mathematical pilgrim as he carefully picks his exciting way, and discovers for himself the basic results in linear transformations, quadratic forms in n variables, finite differences and other topics, unaware of the work already done in these fields. Young Ramaswami is puzzled when he finds that every linear transformation *in plano* leaves three points unchanged, for in the case of rigid motion there can obviously be but one; a little calculation and he is

delighted to find the paradox dissolving in the presence of the circular points. In another place he notes how one may easily be led to overestimate a result because he has had obtained it by a difficult method. Even in his later work there are exclamations and remarks indicative of his delight at discovering a particularly elegant result.

To Ramaswami Aiyar mathematics was not a mere hobby but a part of his very being. When the routine duties of the day were over, he returned to mathematics with something of the surrender and the ecstasy with which a child approaches its mother or a young man his bride. Is it any wonder that his most intimate thoughts and his poignant griefs should find a place in these note-books into which he had poured himself out? that the first should bear the dedication "This book, the first fruits of my mathematical studies I dedicate to my beloved mother to whose tender care and boundless affection for me it is that I owe anything good in me; and I dedicate it with a prayer that she may be my guide and life and light for long long years to come"? And when about ten years later his well-beloved wife "passed away in peace after a few days fever which seemed but ordinary till the morning of her death" the distracted husband can conceive of no more appropriate tablet to her dear and sacred memory than the fly-leaf of the latest book which was the repository of his mathematical communings. He breathes a prayer that her dear and sweet memory may remain fresh to cheer him up in Life and Duty, and breaks out with the cry which has echoed emptily through the ages: "Soul of inexpressible Beauty, where art thou?"

Ramaswami Aiyar's earlier work

Some of the questions discussed in the quarto volumes were those proposed in the Educational Times, but mostly they are either formulated by himself or generalisations suggested by questions in text-books. They cover a wide variety of topics, though geometrical problems predominate.

The very first problem relates to the locus of the incentre I of a quadrilateral A B C D the sum of whose pairs of opposite sides are equal, when one of the sides A B is held fixed. It is shown that if A B is divided at P in the ratio of the adjacent sides, then $PI = \sqrt{(\alpha\beta\gamma\delta)}/s$ where $\alpha, \beta, \gamma, \delta$ are the lengths of the sides and $s = \alpha + \gamma = \beta + \delta$, so that the locus of I is a circle centre P. The corollory is drawn that in a jointed quadrilateral like the one above, if each side is divided in the ratio of the adjacent sides, the four

points of division are concyclic, and the radius of the circle is independent of the form of the quadrilateral.

Centres of mean position

A concept used frequently is that of the masses (multipliers) λ_r for which the centre of mean position of the points A_r (x_r, y_r, z_r) becomes indeterminate, *i. e.* for which

$$\sum\lambda = 0 \quad \sum\lambda x = 0 \quad \sum\lambda y = 0 \quad \sum\lambda z = 0.$$

For such a system, the moment of inertia about any two parallel lines is the same. The following results are stated about such systems

(i) If the moment of inertia about every axis is positive, then P being any point whatever, $\sum\lambda_r \text{ PA}_r > 0$.

(ii) For any point P arbitrarily chosen, $\sum \text{PA}_r^2$ is the same. Taking P at A_1, A_2...successively and multiplying by λ_1, λ_2... we get $\sum\lambda_r\lambda_s(rs)^2 = 0$ where (rs) is the distance $P_r\,P_s$.

(iii) By eliminating λ_r between the equations mentioned above and $\sum\lambda_r = 0$, the identical relation between the mutual distances of four points in a plane is proved, namely

$$\begin{vmatrix} 0 & (12)^2 & (13)^2 & (14)^2 & 1 \\ (21)^2 & 0 & (23)^2 & (24)^2 & 1 \\ (31)^2 & (32)^2 & 0 & (34)^2 & 1 \\ (41)^2 & (42)^2 & (43)^2 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{vmatrix}$$

If the four points lie on a circle, it is shown that the unbordered determinant also vanishes.

(iv) If P is the mean centre of 3 points in a plane (or $n+1$ points in n-space) for multiples λ_r whose sum is unity and O is the circumcentre and R the circumradius, then $R^2 - OP^2 = \sum\lambda_r\lambda_s(rs)^2$. If in a triangle the line IG is produced to K so that $GK = 2\text{ IG}$, then the multipliers for K are $1 - \frac{a}{s}$, $1 - \frac{b}{s}$, $1 - \frac{c}{s}$ and the above result gives on simplification $OK^2 = (R - 2r)^2$. This proves Feurbach's Theorem.

(v) In an analogous manner let θ_1, θ_2, ... be the circular measures of the arcs from a fixed origin O on a circle to n points $A_1\ A_2 \dots A_n$ on the circle. If $\lambda_1\,\lambda_2$...be such that $\sum\lambda_r \cdot \cos\frac{1}{2}\theta_r = 0$ and $\sum\lambda_r \sin\frac{1}{2}\theta_r = 0$, these may be called the multipliers for

which the points have *no circular mean centre* and if P be a variable point on a circle, then $\sum \lambda\, OA = 0$. (chord OA)
From the definition, can be found the *circular mean centre* of n given points on the circle for given multiples. Hence the result: If P is a variable point and A_1 A_2...are fixed points on a circle, then $\sum \lambda_r\, PA_r$ varies as the distance of P from a fixed point on the circle. Ptolemy's Theorem is a particular case of the above.

Homographic Figures in perspective

A little later the author discusses whether two n-dimensional figures in homographic (projective) relation could be so placed as to be in perspective. He finds that for this there should be two linear $n-1$ spaces one belonging to each figure which are "equal in all respects", that is, between which the homography reduces to a congruent transformation, and that when they are made to coincide point for point, the figures will be in perspective. For two plane figures, he finds that such lines of equality exist, there being two in each figure both parallel to the vanishing line in the plane, and so the problem is soluble; but for $n>2$ the problem is insoluble*. In 3-space he finds on each plane parallel to the vanishing plane a double system of parallel "lines of equality", but the surface generated by all the lines meeting a given line perpendicular to the vanishing plane is not a plane but a "double cylindroid" of the form $(x^2+y^2)\,z^2 = a^2x^2+b^2y^2$. If $a^2=b^2$, this includes two planes $z^2=a^2$ and only then can the figures be placed in perspective.†

Brocard motion and harmonic polyhedra

A study of Casey's** treatment of harmonic polygons suggested to Ramaswami Aiyar an extension to polyhedra. If a particle describes a circle in time t with a velocity varying as the distance from a fixed line L, it is called "Brocard motion". If two particles describe a circle with Brocard motion their join envelopes an ellipse having

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* The general linear transformation group in n-space is one with $(n+1)^2-1$ parameters while perspectively (determined by a point, a prime and a cross ratio) contains $2n+1$ constants and a displacement $n\,(n+1)/2$ constants. Hence $(n-2)\,(n+1)/2$ further conditions have to be fulfilled by the figures if they can be placed in perspective position.

† In geometrical language, this is equivalent to the statement that the imaginary circle at infinity should transform into a *circle* in the vanishing plane. Vide: *Encyclopadie Math. Wiss. III I* (i) p. 426. The corresponding requirement in n-space imposes exactly $(n-2)\,(n+1)/2$ conditions.

** Casey: *Treatise on the Analytical Geometry of the point, line, circle and conic sections* (1893) Chapter XIV Section, II.

double contact with the circle along L – the Brocard Ellipse. If n particles describe a circle with Brocard motion and be distant apart by an interval of time t/n, their instantaneous positions form the vertices of a "harmonic polygon." It is shown that Brocard motion is the inverse of uniform circular motion and it thus follows that a harmonic polygon is the inverse of a regular polygon and that the vertices of a harmonic polygon invert, with respect to any point in space, into those of a harmonic polygon. If two particles in Brocard motion be apart by an interval of time $t/2$, their join passes through a fixed point k, the Lemoine point of the motion and of the polygons. This point has the characteristic property that its distance from the sides of a harmonic polygon are proportional to the lengths of those sides.

The inverses of the vertices of a regular polyhedron are said to form those of "harmonic polyhedron." Its faces are harmonic polygons and their Lemoine lines lie on a plane—the Lemoine plane of the polyhedron. The pole of this plane w. r. t the circumsphere or w. r. t the polyhedron is the Lemoine point k. The limiting points of the Lemoine plane as radical plane and the circumsphere are two points from which the polyhedron inverts into a regular polyhedron. If the joins of k to the vertices be divided in the same ratio and planes drawn parallel to the tangent planes at the vertices to the circumsphere, these cut out of the respective solid angles equal regular polygons all of whose vertices lie on a sphere and whose sides touch another sphere. The envelope of each of these systems of Tucker spheres is an ellipsoid of revolution, the first touching the edges and the second the faces of the polyhedron. A tetrahedron is harmonic only if the product of its pairs of opposite edges is the same. These results are extended to n-space.

For any general tetrahedron ABCD sections $\alpha, \beta, \gamma, \delta$ parallel to the tangent planes to the circumsphere cut out similar triangles out of the solid angles at A, B, C, D. If corresponding points a, b, c, d are taken in these triangles, the joins Aa, Bb, Cc, Dd are the generators of a hyperboloid and if a′ b′ c′ d′ are the isogonal conjugates of a, b, c, d in the sections, Aa′, Bb′, Cc′, Dd′ are generators of the opposite system of the same hyperboloid. It follows that if a, b,... are the in- or ex-centres of the sections, the joins Aa, Bb,...etc. are concurrent. There are thus four points of concurrence H_1 H_2 H_3 H_4 which Ramaswami Aiyar calls the "Casey points" of the tetrahedron "in memory of him who opened to me the world of geometry." The tetrahedron H_1 H_2 H_3 H_4 is in desmic relation to ABCD. When H

is any of these points and the sections $\alpha, \beta, \gamma, \delta$ mentioned above are drawn so as to divide HA, HB, HC, HD, in the same ratio, the sides of the four equal triangles thus obtained touch a "Casey Sphere". The envelope of the Casey spheres is an oblate spheroid touching the faces of the tetrahedron. When the tetrahedron is "harmonic", the internal Casey point and corresponding spheres become identical with the Lemoine point and the second Tucker system of spheres.

Later Work

I shall refer briefly to two topics discussed in the folios, "Self reversible functions" and "The Durairajan Point."

Self Reversible Functions

We know that if $x' : y' : z' = yz : zx : xy$ then $x : y : z = y'z' : z'x' : x'y'$. Ramaswami Aiyar applies the term "self reversible" to a homogeneous function $\phi(x, y, z)$ if from

$$x' : y' : z' = \phi(x, y, z) : \phi(y, z, x) : \phi(z, x, y)$$

it follows that

$$x : y : z = \phi(x', y', z') : \phi(y', z', x') : \phi(z', x', y')$$

Representing (x, y, z) and $(x'\, y'\, z')$ by points in the same plane, it is seen to be the study of involutoric Cremona transformations which are permutable with the cyclic collineation $x' : y' : z' = y : z : x$.

It is shown that if $\phi_1(x, y, z)$ and $\phi_2(x, y, z)$ are cyclically symmetric in x, y, z and satisfy the relations $\triangle^3\phi_1 \equiv 0$, $\triangle\phi_1 \equiv 2\phi_2$ where $\triangle$ stands for $\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}$, then from

$$x - x' = y - y' = z - z' = \phi_1(x, y, z)/\phi_2(x, y, z)$$

it follows that

$$x' - x = y' - y = z' - z = \phi_1(x', y', z')/\phi_2(x', y', z').$$

We thus have $\phi = x\phi_2 - \phi_1$ as a "self reversible" function. Thus from $\phi_1 = \sum yz$, one may obtain the self reversible transformation $x' : y' : z' = x^2 - yz : y^2 - zx : z^2 - xy$.

Geometrical considerations are also pressed into service and Ramaswami Aiyar lists a large number of self reversible functions of which a few are given below:

$$\phi = yz; \quad \phi = x^2 - yz, \quad \phi = x(y + z - x)$$
$$\phi = \alpha(\sum x^3 - 3xyz) + \beta(\sum x^2 y + \sum x y^2 - 6xyz)$$
$$+ \gamma(y - z)(z - x)(x - y) - (\tfrac{3}{2}\alpha + \beta)\, x(\sum x^2 - \sum yz)$$

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* For the properties of harmonic and other tetrahedra reference may be made to :
(1) Altshiller-Court—*Modern Pure Solid Geometry Ch. IX.*
(2) Couderc & Balliccioni : *Prèmiere livre du Tètraèdre* (1935)

provided $\frac{3}{2}\alpha+\beta\neq 0$. This includes all reversible functions of the third order

$$\phi=(x-y)(x-z)(l\,y\,z+m\,z\,x+n\,x\,y) \text{ if } l+m+n=0$$
$$\phi=(2\,x-y-z)(2\,z\,x-x\,y-y\,z)(2\,x\,y-y\,z-z\,x)$$
$$\phi=(x-y)^2(y-z)\,z\,x$$
$$\phi=\{x(y-z)-\theta\,z(x-y)\}\,\{y(z-x)-\theta\,x(y-z)\}\,\{(z-x)-\theta(x-y)\}$$

The Durairajan Point of a quadrangle

In Vol. 17 the *Journal of the Indian Mathematical Society* (August 1927), the following question was proposed by Mr. N Durairajan:

"A, B, C, D are four points in a plane. A′ B′ C′ D′ are respectively the isogonal conjugates of A, B, C, D with respect to the triangles BCD, CDA, DAB, ABC. Show that there exists a point M such that A′ B′ C′ D′ are the inverses of M with respect to the circles BCD, CDA, DAB, ABC."

The point M has been named "the Durairajan point of the quadrangle" by Ramaswami Aiyar and has been the subject of close study by him during the last two years and earlier by Mr. A. A. Krishnaswami Ayyangar. The results obtained were communicated to the *Student* from time to time and some unpublished results appear as questions at the end of this issue. The following questions in the *Journal* and the *Student* relate to the Durairajan point:

Questions: 1480, 1481, 1485, 1495, 1698, 1710, 1711, 1720, 1721. The Durairajan point is invariantly related to the points A, B, C, D under the inversion group and may be obtained by circle constructions. It is perhaps most elegantly defined as the transform of the unique point at infinity in the tetracyclic plane in Dr. Vaidyanathaswami's generalised isogonal transformation* defined by the tetrad of point circles A, B, C, D.

The topics detailed above are only a selection, almost at random, from among those discussed in the note-books. An exhaustive study of Ramaswami Aiyar's researches and its relation to present-day knowledge in the various topics would be a task involving patient work over a considerable period. I hope, however, that this will be undertaken and a carefully edited edition of Ramaswami Aiyar's contributions to mathematics brought out with the co-operation of the Madras University, of which he was a distinguished *alumnus*, and of the Indian Mathematical Society which he brought into existence. Such a publication should be of help not only to research workers

* Vaidyanathaswami: *A Memoir on cubic transformations associated with a desmic system*: Suppl. to Vol. XV of the *J. I. M. S.* p. 54.

but also to college students and teachers, for Ramaswami Aiyar uses elementary methods with telling effect and has the master's secret of leading his readers on through intricate mazes without their ever feeling that their journey has been other than simple and straight. Has not the Tamil proverb declared that "In the hands of the mighty even a blade of grass is an effective instrument."?

ASTRONOMICAL NOTES FOR JANUARY AND FEBRUARY 1937

The Sun. After the Winter Solstice on Dec. 22nd, the apparent motion of the Sun has been north-easterly. Its motion among the constellations would be from Sagittarius to Capricornus, and towards the end of February it will be on the north-eastern part of Aquarius. The position of the Sun on the 1st of January and February will be respectively as follows :—

R. A. 18h-46m ; Dec. 23-2′ south ; and R. A. 20h-58m Dec. 17-12′ south.

The Moon. The following phases of the Moon will occur on the dates specified.

	January.	I. S. T.	February.	I. S. T.
		H. M.		H. M.
Last quarter.	4th	19-52.	3rd	17-34.
New Moon.	12th	22-17.	11th	13- 4.
First quarter.	20th	1-32.	18th	9-20.
Full Moon.	26th	22-45.	25th	13-13.
Apogee.	6th	20-30.	3rd	17-30.
Perigee.	22nd	8-30.	16th	1-30.

Mercury. Mercury will be moving with an apparent easterly motion. On January 5th it will be stationary and will have the inferior conjunction with the Sun on the 14th January. Again it will be stationary on January 26th. It will reach its greatest elongation west on February 7th. Its position on 1st January will be at the point of R. A. 20h-8m, Dec. 20°-55′ S.

Venus. Will be moving from the constellation Capricornus to Aquarius and Pisces and it will be an evening object. On

January 16th it will be in conjunction with the Moon and on 24th January with Saturn. It will be at its greatest elongation on February 5th.

Mars. Will be in the constellation Libra. Its conjunction with the Moon takes place on 6th January and 3rd February. Its position on the 1st of January will be at the point of R. A. 13h-44m and Decli. 9°9′ south.

Jupiter. Will be a morning object in Sagittarius. Conjunction with the Moon on 11th January and 8th February. Its position on the 1st January is given by R. A. 18h-30m and Dec. 23°-12′ south; and on 1st February by R. A. 19h-0m and Decli. 22°-43′ south.

Saturn. Will be an evening object in Aquarius. Its conjunction with the Moon occurs on the 17th January and the 13th February. Its distance from the Earth will be about 900 million miles. Its position on 1st January is given by R. A. 23h-17m; Decli. 6°-54′ south; and on 1st February by R. A. 23h-27m; Decli. 5°-45′ south.

Uranus. Will be stationary on the 14th January and is in Aries. Conjunction with the Moon occurs on 20th January and 16th February. Its motion among the stars will be retrograde.

Neptune. Will be a morning object in Leo. Its distance from the Earth will be about 2800 million miles and its apparent diameter will be about 2·4 seconds of arc.

Comet Notes. It is reported that the third Comet of 1936 (Comet 1936 C) was discovered by Mr. C. Jackson of Johannesburg, South Africa, on Sept. 21st. It was detected on plates taken on Sept. 15th for minor planet observations. It was of the 12th magnitude. It was near its maximum brightness at the time of discovery and since then it has faded rapidly. Its period of rotation has not yet been definitely fixed, but it has been roughly estimated to be about 7 or 8 years.

New Variable Stars in Northern Milky Way. In the investigations by the Staff of the Harvard Observatory on the size of the galactic system and on the distribution of obscuring matter within its bounds, the discovery of 27 new variables in the region of the Milky Way opposite to the direction to the centre in Sagittarius, was reported. The region studied is in

Auriga. The long-exposure photographs revealed that external galaxies are very numerous in that region. The new variable stars in Auriga include several types: long period, irregular, and cluster-type Cepheids predominating.

New Stars. A Nova in the constellation Aquilla (Nova 668/1936) was found by Prof. Tamm from examination of photographic plates taken at the Stockholm Observatory. Its magnitude in October was 7·6 and its position is at R. A. 19h-23m, Decli. 7-29′ north. Similarly another Nova was also discovered by him on 18th Sept. It was of 8th magnitude and in the constellation Aquilla. Its position is R. A. 19h-14m and Decli. 1-36′ N. Mr. C. Jackson of Johannesburg has confirmed the discovery of a Nova in Sagittarius by an observer in Tokyo. It was of the 6th magnitude in October.

The prominent constellations that could be observed during January and February are the following :—

Zodiacal constellations from west to east consist of the following in order. Sagittarius (the Archer), Capricornus (the Sea Goat), Aquarius (the Water-Bearer), Pisces (the Fishes), Aries (the Ram), Taurus (the Bull) and Gemini (the Twins). Of the other star groups Pegasus, Cygnus, Andromeda, Cassiopeia, Auriga, Canis Major, Canis Minor, Orion, are some of the prominent ones.

P. S.—The positions given are for the mean noon of Trivandrum or 0h-22m P. M. of Indian Standard Time.

H. SUBRAMANI AIYAR,
Prof. of Mathematics and Government Astronomer,
Trivandrum.

Nova in Pleiades ?

Dr. H. Parameswaran, Prof. of Physics, Presidency College, Madras claims to have discovered what appears to be a Nova in Pleiades. As recorded on a plate exposed on 4-12-1936, it is approximately of the 7th magnitude and later exposures of the region on fast plates even for 15 minutes show only a few stars of probably the 10th magnitude. There is a slight nebulosity surrounding the nucleus and a suspicion of displacement which suggests that it might be a receding comet.

A. N. R.

SOLUTIONS TO QUESTIONS

Question 1650

(V. RAMASWAMI AIYAR) :—Two triangles ABC and A′ B′ C′ are in perspective. If any conics whatever circumscribe the triangles and pass through the centre of perspective O show that they cut again in three points which form a triangle self conjugate with respect to a fixed conic.

Solution by K. Rangaswami

It is well known that the lines and planes determined by five points in general position in space meet a given plane p in a system of ten points and ten lines forming a Desargues' 10_3 configuration. Further this system is self polar with regard to the Unique Conic σ inpolar to the five linearly independent conics in which the plane p meets any five linearly independent quadrics through the five points.

Let a cubic curve through the five points in space meet the plane p in P, Q, R. Now, since every quadric through the five points is outpolar to σ, the net of quadrics through the cubic meet the plane p in a net of conics circumscribed to PQR and also outpolar to σ.

Further as we can project the cubic from any *two* of the five points in space into conics through OABC; OA′B′C′ taking the points of the plane in this particular order it follows that the ∞^2 triangles PQR cut out by any conic through OABC and OA′B′C′ are the triangles determined on p by the ∞^2 cubics through the five points of space and are, therefore, self conjugate with regard to σ.

Question 1687

(V. RAMASWAMI AIYAR AND A. NARASINGA RAO) :—Prove that perpendicular tangents to any parabola inscribed in an equilateral triangle are also principal axes of a conic inscribed in its medial triangle.

Question 1689. (Corrected).

(V. RAMASWAMI AIYAR AND A. NARASINGA RAO) :—ABCD is an orthocentric tetrad of points and l is any straight line. Parabolas are drawn inscribed in each of the triangles ABC, BCD, CDA, DAB and touching l. Show that all the parabolas touch another line l' and that l and l' are the principal axes of conics inscribed in the pedal triangle of ABCD.

Solution by K. Satyanarayana

We know that the foci of a conic for which a triangle is self-polar are isogonal conjugates with respect to the pedal triangle and hence are also the foci for some in-conic of the pedal triangle.

Consider μ the in-conic of the pedal triangle of ABCD having l as a principal axis. Let l' be the other principal axis.

Let μ_1, μ_2, μ_3, μ_4 be the conics, confocal with μ, for which the triangles BCD, CDA, DAB, ABC, are respectively self-polar.

Now for μ_1, BCD and the triangle formed by l, l' and the line at infinity are both self polar and hence the six sides of the triangles touch a conic.

Hence the parabola inscribed on ABC and touching l, and similarly the other parabolas touch l' and the result in question 1689 follows.

As regards Question 1687, take ABC as equilateral. Then the pedal triangle becomes the medial triangle and the result is obvious from the previous argument.

Also solved by A. Ranganatha Rao, T. K. Raghavachari and K. Rangaswami.

Question 1690

(V. RAMASWAMI AIYAR) :—A conic passing through the in- and ex-centres of a triangle ABC has one asymptote parallel to BC. Show that that asymptote passes through A and the other asymptote passes through A′, the point diametrically opposite to A on the circumcircle of ABC. If DEF be the medial triangle of ABC, show that the conic passes through D, and the tangent to it at D is the median AD. Show that the conic also passes through the foot of the perpendicular from A on EF; and that the tangent to it at the point passes through the foot of the perpendicular from A′ on BC.

Solution by A. Ranganatha Rao.

The conic in question is necessarily a rectangular hyperbola Γ, whose centre O lies on the circumcircle of ABC, and for which ABC is a self-polar triangle. Hence, Γ divides BC harmonically, and as one common point is at infinity, the other is the mid-point D of BC. Since the polar of A is BC, the tangents to Γ at D and the point at infinity on BC pass through A; the latter tangent is thus an asymptote. The other asymptote being perpendicular to this, must

pass through A′. If AP is drawn perpendicular to BC, A and P are conjugate points and AP is parallel to the asymptote OA′. Thus, as before, Γ cuts AP at its mid-point which is the foot of the perpendicular from A on EF. If A′O meets BC at K, the polar of A passes through K and so the polar of K passes through A. Hence AP is the polar of K. Therefore the tangent to Γ at the mid-point of AP passes through K.

Also solved by K. Satyanarayana, T. K. Raghavachari, S. Narayanan and K. Venkatachala Iyengar.

Question 1703

(V. RAMASWAMY AIYAR) :—Prove that the nine-points circle of the diagonal triangle of any quadrilateral passes through the miquel point of the quadrilateral.

Solution by V. Seetharaman

Consider the parabola touching the four sides of the quadrilateral. The Miquel point is the focus of this parabola. The diagonal triangle is self conjugate with respect to the parabola. Hence its medial triangle is circumscribed to the parabola. Therefore the circumcircle of the medial triangle i.e. the Nine-points circle of the diagonal triangle, passes through the focus of the parabola.

Also solved by T. K. Raghavachari, M. K. Srinivasa Rao, S. Narayanan, K. Naghabhushanam, C. N. Srinivasa Ayyangar and K. Rangaswamy.

GLEANINGS

46. "It used to be narrated at Cambridge that Todhunter once dreamt that he was minus one under a root and could not extract himself."

Prof. G. H. Bryan's Presidential Address before The Mathematics Association 1908.

C′ est de l′ algèbre

47. J' obtins un jour de mon adolescence, un prix d' algèbre. De doux commeres qui me felicitaient, l' une demande "Mais qu' est-ce que c'est que l' Algèbre?"—"C' est la langue qu' on parle en Algerie".

Sphinx: Aug. 1936.

REVIEWS

GEORGE ABRAHAM MILLER: *Collected Mathematical Works, Volume I.*

[University of Illinois, Illinois, 1935, pp. 475, price $ 7.50.]

The mathematical public is greatly indebted to the University of Illinois, for undertaking to publish the collected works of Professor G. A. Miller, as a fitting memorial of his services in the field of mathematics and to the University. For over forty years, the name of Prof. Miller has been closely associated with the development of the theory of groups, an important branch of modern mathematics, which, although it had its origin in the theory of equations, was later on found to be of such fundamental importance that Poincare was led to remark "the theory of groups is, so to say, the whole of mathematics divested of its matter and reduced to pure form".

The present volume which is the first, comprises 62 papers, being Prof. Miller's contributions up to 1900. With the exception of two interesting digressions in papers 12 and 60 dealing with Lie's views on several important points in mathematics, and some remeniscences in regard to him, the rest relate to the theory of finite groups. In papers 1, 15 and 62, which were specially written for this volume, we have a lucid and masterly presentation of the history of the subject up to 1900, and the place of his own work in the theory.

The success which attended the application of the theory of substitution groups, by Abel and Galois in their researches in the theory of equations led to an increasing interest in the subject. One of the most important problems, interest in which was further stimulated by the institution of prizes by various Academies was that of determining all the groups of a given degree n. Serret succeeded in enumerating all the 19 groups of degree 5. Cauchy, Cayley, Kirkmann and later on Askwith attempted the case of $n=6$. While Kirkmann was correct in the determination of the 16 transitive groups of this degree, Askwith was correct as regards the 21 intransitive groups of the same degree, F. N. Cole succeeded in determining all the 40 groups of degree 7 and attempts for a higher degree were incomplete. The problem increases in difficulty with the degree, and credit for further work is due entirely to Miller, who coming under the influence of Cole, tackled the problem with indomitable courage and skill, and succeeded in enumerating the 200 groups of degree 8, the 258 groups of degree 9 and the 1039 groups of degree 10. His methods could not carry him further, and he had to restrict himself to special types of a higher degree, like the transitive groups of degree 12, 13, 14 and 17, and the primitive groups of degree 15 and 16. These topics cover roughly a third of this volume. These achievements, admirable by themselves, do not, however, take us a step further to the solution of the general problem for any degree which remains as difficult as it was a century back.

From the above point of view, it is gratifying to note, that a greater measure of success had attended the solution of the parallel problem of enumerating

groups of a given *order*. The problem is known after Cayley, who initiated it and gave a correct determination of the 5 groups of both of the orders 8 and 12. Many were working at it, after Cayley, and before Miller entered the field groups of order up to 24 were known, and Netto had discovered two remarkable general theorems, the first one being, that there are only two groups of order p^2 when p is prime, and the second that there are also only two groups of order pq where p and q are primes such that $p \equiv 1 \mod q$. Miller worked out the groups of order up to 64, and also groups of some general orders viz. those of order pq, pqr, $8p$, p^3, $2p^3$ where p, q, r are arbitrary primes. These form the topics of about dozen papers. Of these the investigations for the cube of any prime and later researches by Young and Hölder for fourth and fifth powers are of importance towards the solution of the general problem, especially in virtue of Sylow's Theorem.

The remaining papers are all contributions to the theory of abstract groups. It was Cayley who in 1878, took up the abstract point of view, when he recognised that the group is defined purely by the laws of combination of its symbols and is independent of the concrete representation of these symbols. This led to a re-examination of the theorems in the theory of substitution groups, with the result that some of them could be taken over to the abstract theory and some others led to the formulation of further problems in the new theory. One such is the definition and study of systems of solvable groups, which was suggested by substitution groups associated with algebraic equations solvable by radicals. Paper 37 is devoted to an important infinite system of solvable groups viz. the Hamiltonian system, which is the generalisation of the quaternion group, and one remarkable theorem is that there is only one Hamiltonian group of order $2a$ when $a > 2$. Meanwhile Lie, who had been developing the theory of continuous groups, suggested the study of the derivatives of finite groups also (The first derivative of G is the sub-group formed by all the commutators of G.) This was taken up by Miller in papers 18, 34 and the most important theorem is, that the necessary and sufficient condition for the solvability of a group is that we should arrive at identity by forming its successive derivatives. Paper 39 is devoted to perfect groups, that is, groups identical with their first derivatives and in paper 36, we have an extension of Sylow's theorem.

It is not possible to go into further details. Suffice it to assure any reader, with some mathematical insight and maturity of mind, that it will be a delight for him to delve into these pages and follow the working of a master mind in this subject. The remark of Poincare, given in the opening paragraph, was made when the group concept had permeated all mathematical theories. To-day it has also entered regions where mathematics is applied. Not only the group concept, and some obvious statements about it, but also one of the deepest part of group theory dealing with the representation of groups by linear transformations is found to be of paramount importance in Quantum Mechanics. Under these circumstances, it is highly regrettable to note that even the elements of the theory have not found a place in the curriculum of study of many Indian Universities. Let us hope, that the appearance of this inspiring volume, so finely got up, opening with an arresting portrait of the author beaming with youthful enthusiasm, and a subject index and an author index at the end, and the remaining volumes to be published, will serve

to shake off our lethargy and create and stimulate our interest in what Eddington calls this super-mathematics, where we neither know what we are talking about nor what we are doing with them.

16-12-36 B. RAMAMURTI.

WEBSTER: *Partial Differential equations of Mathematical Physics*: Edited by S. J. PLIMPTON G. E. Stechert & Co., New York and B. G. Teubner Leipzig. pp. 440

In the last decade of the last century Webster published a book under the above title which was considered very helpful to mathematical physicists. It may surprise many to know that it was one of the earliest English books in which correct definitions of the fundamental notions such as limits, continuity and differentiability were given. Prof. Plimpton brought out in 1927 the second revised edition of the book after the death of Webster. The present one is a reprint of the former except for minor changes. At about the same time Szego brought out a German edition of Webster's book. Both the books are almost identical.

The two standard treatises on the subject are—Frank-Mises—*Partielle Differentialgleichungen der mathematischen Physik,* 2 volumes [which is the seventh edition of Riemann's book revised by Weber.] and the magnificent and very original treatise Courant-Hilbert—*Methoden der Mathematischen Physik*. The former written by a number of distinguished specialists in the various branches of the subject is to be looked upon as a book of reference by research workers and is encyclopaedic in its scope. The latter does not deal with all the topics of mathematical physics but is chiefly concerned with the methods—or rather the correlation of the present-day guiding ideas of mathematical physics with those of Mathematics as is emphasised by Courant in the preface.

The book under review is more akin to the former treatise and forms a very natural introduction to it. The book covers fully everything in this field, and as it assumes a minimum of previous knowledge on the part of the reader, it should appeal equally to a physicist as well as a mathematical student. There is one more point of interest about the book specially to lecturers in our universities. The present-day syllabus in differential equations of most of our Universities cannot be correlated with either the theory of differential equations or mathematical physics. This book being in English, portions of it can very easily fit themselves into the syllabus of our courses.

The various chapters in the book are the following—Deduction of differential equations—Equations of first order—Wave equation, vibrations and normal functions—Fourier's series and integral, Cauchy's method, Initial data—Methods of green, Potentials—boundary problems—Methods of Riemann-Volterra—characteristics—spherical, cylindrical and ellipsoidal harmonics—Applications of the preceding—Theory of Integral equations—Appendix: Proofs of some of the mathematical results assumed.]

K. V. I.

MANDELBROJT, *Series Lacunaires*: Actualitiés Scientifiques et Industrielles, No. 305, Herman & Co., Paris pp. 40, price 12 fr.

This short Monograph by Prof. Mandelbrojt gives a summary of recent researches dealing with the effect of gaps in power (or Dirichlet) series on the existence and nature of the singularities of an analytic function. He considers a singularity in the most general sense. Thus in the case of an integral function the existence of a singularity is replaced by a Julia-direction i.e. a straight line issuing from the origin which is such that in any angle (with the origin as vertex) the function takes every value with the exception of at most one value.

The proof of the main theorem is founded on two ideas of Faber who gave 30 years ago a very interesting and original proof of Fabry's gap theorem. [Bieberbach-*Lehrbuch der Funkhionentheorie* Bd. II]. By means of this the author obtains remarkable precisions and generalisations of many of the earlier theorems. It is not possible to state in the course of the review, all the results contained in the tract. This result due to Ostrowski is generalised in the book—If $f(z) = \sum d_n \exp(-\lambda_n z)$ where $\underline{\text{Lim}}\ \lambda_n/n = G > o$. $\underline{\text{Lim}}\ (\lambda_{n+1} - \lambda n) = h > o$, where $f(z)$ possesses an axis of convergence, then $f(z)$ has a singular point in every circle whose radius is equal to $[\pi + 3 \log (e^6 h^{-1} G)]/G$ and having its centre on the axis of convergence. The last chapter contains results on the influence of gaps on the nature of the singularities, the singularities considered being poles and algebraic-Logarithmic singular points. The book should prove very helpful and stimulating to a student of the theory of functions as much of what is contained therein has not as yet appeared in book form.

K. V. I.

J. FAVARD; *Les theoremes de la Moyenne de Polynomes*

Actualities Scientifique et Industrielles No. 302, Herman et Co., Paris; pp. 48, Price 15 frs.

This short monograph deals with some of the results on the location of roots of polynomials whose coefficients are connected by a linear relation. In the first chapter the author gives results connected with Grace's theorem for which he gives two proofs, the first being based on the theory of normal functions which does not give us the precise constant but is capable of generalisation, and the second a modification of the beautiful proof by Szego [See Polya and Szego—*Aufgaben und Lehrsätze*—Bd II. Ab. V]. The second chapter deals with the general problem and generalisation to polynomials in two variables. Much of what is contained in the book and the results in Polya-Szego [loc. cit.] ab. V are not difficult and might well be incorporated in the syllabus of the theory of equations for Honour's courses in our universities.

K. V. I.

ANNOUNCEMENTS AND NEWS

Prof. S. B. Belekar and Rai Saheb Mukund Lal have retired from the committee of the Indian Mathematical Society by efflux of time. Dr. P. L. Srivastava, M.A., D.Phil. (Oxon.), Reader, University of Allahabad and Dr. S. Chowla, M.A., Ph.D. (Cantab.) Professor, Government College, Lahore, have been elected and have consented to serve on the committee. The composition of the reconstituted committee is now as follows:

Dr. R. P. Paranjpye: Vice-Chancellor, Lucknow University, *President.*
Prof. L. N. Subramaniam, Madras, *Treasurer.*
Prof. R. P. Shintre, Poona, *Librarian.*
Dr. R. Vaidyanathaswami, Madras and Dr. Ram Behari, Delhi, *Secretaries.*
Dr. Mahajani, Poona.
Principal T. K. Venkataraman (Retd.), Kumbakonam.
Prof. Arunachala Sastri, Hyderabad.
Prof. Narasinga Rao, Annamalainagar.
Dr. Mukhopadhyaya, Calcutta.
Dr. P. L. Srivastava, Allahabad.
Dr. S. Chowla, Lahore.

The following gentlemen have been admitted as members of the Society:

Sundararama Sastry Esq., M.A., M.Sc., Lecturer in Statistics, University of Madras.

Satya Swaroop Esq., M.A., All-India Institute of Hygiene and Public Health, Calcutta.

The second International Congress of Mathematical Recreations will be held in Paris in July 1937 in one of the stands of the Exposition. Several advantages are offered to congress members—reduction in train fare, travel facilities, free entry into the Exposition etc.

The Madras University announces that the Ramanujam Memorial Prize (value Rs. 500) will be awarded for the best essay or thesis written on any branch of Mathematics, embodying the result of personal investigations of the author and containing clear evidence of independent and original research. The Prize is open to all persons born or domiciled in India. Essays should be forwarded not later than Dec. 1st, 1937 to the Registrar.

A cash reward of $ 1000 is offered by the Williams and Wilkins Co., Baltimore for the best manuscript on a Science subject, presented before July 1st 1937. No limitation is placed on the subject-matter or manner of handling and more on the eligibility of the award. The manuscript must be in English and "of a sort calculated to appeal to the taste of the public at large". The desired length is given as 100,000 words.

Mr. V. Ramaswami, Reader and Head of the Department of Mathematics, the Andhra University, Waltair, has been awarded the Ph.D. Degree by the Cambridge University. Mr. Ramaswami passed the Tripos with distinction

from Kings College, Cambridge, and carried out research work under the guidance of Prof. Littlewood. The title of the main thesis is "Some Tauberian Theorems on Oscillation."

The Indian Mathematical Society acknowledges with thanks the financial help it has received from the Annamalai University (Rs. 100 for the year 1936-37) and the University of Madras (Rs. 150 per annum for three years commencing from 1936-37).

The following periodicals are being received as Exchanges or as free gifts:

1. Publications de l'Institut Mathématique de Strasbourg.
2. Wiadomosci Matematyczne.
3. Studia Mathematica.
4. Fundamenta Mathematicae.
5. Journal of Mathematics and Physics of the Massachusetts Institute of Technology.
6. Arkiv für Matematik Astronomi och Fysík.
7. Proceedings of the Indian Academy of Sciences. [A]
8. Deutsche Mathematik.
9. Monthly Notices of the Royal Astronomical Society.

The following periodicals have been suspended for the time being:—

1. Proceedings of the Royal Soc. A.
2. Astrophysical Journal.
3. Philosophical Magazine

and in their place the following will be subscribed:—

1. Math. Zeitschrift.
2. Zentralblatt für Mathematik.

OBITUARY

DEWAN BAHADUR R. RAMACHANDRA RAO

The death of Dewan Bahadur R. Ramachandra Rao removes from the Indian Mathematical Society one more of the dwindling number of its Foundation Members who are happily still left to us. He served as a member of the first Committee of the Indian Mathematical Club along with Messrs. T. R. Venkatasami Naidu and R. N. Apte, and later with Mr. Balakram, and one of his first acts was to present the Club with several books and a sum of Rs. 100/- to serve as a nucleus for a library. He was elected President of the Society and served in this capacity between 1915 and 1917.

A member of the short-lived statutory Civil Service, he made his mark early as one of the ablest officers of his time by the vigour, versatility and power of initiative he brought to bear on his work and achieved for himself an enviable place in popular esteem by the patience and understanding with which he studied the wants of the common people and the sturdy independence which characterised all his actions. At a time when the problem of economic uplift

was hardly thought of, he was alive to the importance of increasing the productive capacity of the ryot, and making the country as far as possible self-sufficient, and devoted himself to the cause of Swadeshi and Temperance and the spread of Co-operative ideals. But even more than as an able civil servant and an earnest worker in the cause of national regeneration, his memory will be cherished in the hearts of his fellow-members of the Indian Mathematical Society as the unfailing source from which Ramanujan derived help and encouragement in boundless measure when he was in sore need of both.

A. N. R.

RAO BAHADUR S. NARAYANA AYYAR

We regret very much to report the death of Rao Bahadur S. Narayana Ayyar, retired Chief Accountant, of the Madras Port Trust at the age of 61 at his residence in Madras on the 17th January 1937.

The late Mr. Narayana Ayyar joined Government Service in the Public Works Department in 1900 and was later taken into the Port Trust Office where he served till 1934. He was one of the Foundation Members of the Indian Mathematical Club and served as its first Assistant Secretary from 1907 to 1910 and later as Treasurer of the Society for 14 years from 1914 to 1928. During the whole of these 14 years he rendered valuable service to the Society as a member of its Managing Committee.

In spite of the preoccupations of his official work Narayana Ayyar maintained a keen interest in Mathematics. The following list includes his contributions to the Journal of the Indian Mathematical Society:

Note on $\int \frac{\sin mx}{\sin nx} dx$	Vol.	I	page	9
The distribution of primes	,,	V	,,	60
Some theorems on summation	,,	V	,,	183
Some infinite determinants	,,	VII	,,	51
A small theorem	,,	VIII	,,	181

Questions for solution: Nos. 16, 17, 23, 27, 37, 38, 67, 85, 109, 110, 128, 129, 162, 163, 177, 465, 494, 575, 576, 592, 593, 616, 617, 630, 631, 667, 686, 718, 756, 806, 878, 1394, 1405, 1486, 1500, 1548.

Solutions to questions: 2, 4, 7, 12, 18, 19, 21, 22, 31, 33, 39, 49, 68, 69, 104, 111, 113, 114.

A. N. R.

QUESTIONS FOR SOLUTION

1735. (V. RAMASWAMI AIYAR)—ABCD are four points in a plane and A′ B′ C′ D′ are the isogonal conjugates of A, B, C, D in the triangles BCD, CDA, DAB, ABC. Show that M, the Durairajan point of A, B, C, D is the centre of similitude of six sets of similar figures of which corresponding segments are like (AD, D′ A′) or (BC, C′ B′).

1736. (V. RAMASWAMY AIYAR)—If ABCD be a quadrangle $\alpha, \beta, \gamma, \delta$ the centres of the pedal circles of A with respect to the triangle BCD etc., then

(i) the quadrangles ABCD and $\alpha\beta\gamma\delta$ are inversely similar;

(ii) to the Durairajan point in the former corresponds H (the centre of the rectangular hyperbola through ABCD) in the latter; that is H is the Durairajan point of $\alpha\beta\gamma\delta$.

1737. (V. RAMASWAMI AIYAR)—If a point P and the feet of the perpendicular from it on the sides of a quadrilateral be on a parabola, the locus of P is a Cardioid whose cusp is at the Miquel point of the quadrilateral.

If we have five lines there will be five such cardioids. They have a pair of common points lying on the Miquel circle of the line.

1738. (V. RAMASWAMI AIYAR)—Given a quadrilateral let P be any point and let Γ_p denote the conic passing through P and the feet of the perpendiculars from it on the sides of the quadrilateral.

(i) If Γ_p be a rectangular hyperbola, the locus of P is the centre circle of the quadrilateral.

(ii) The centre circle cuts the circumcircle of the triangle formed by any *three of the lines* in a point whose pedal line with reference to that triangle is parallel to the fourth line.

(iii) If Γ_p be a hyperbola whose asymptotes cut at a given angle the locus of P is a hyperbolic limacon whose node is at the Miquel point of the quadrilateral.

(iv) If Γ_p be an ellipse of given eccentricity the locus of P is an elliptic limacon having a conjugate point (imaginary node) at the Miquel point of the quadrilateral.

(iv*a*) For various given eccentricities or asymptotic angles the limacons form the system $r = a \cos\theta \pm b$ in polar co-ordinates where b varies. The origin is the Miquel point of the system and the circle $r = a\cos\theta$ is the centre circle of the quadrilateral.

(v) Given five lines the centre circles of these taken four at a time are concurrent at a point Z which lies on the Miquel circle of the five lines. The point of concurrence is the focus of the unique rectangular bi-parabola touching the five lines and is also the inverse of the centre of the Miquel circle in the centre circle of the five lines.

(vi) Given five lines five limacons are determined (under iv a) for a given eccentricity or asymptotic angle taking the lines four at a time. The limacons will be similar and they will have the common intersection lying on the Miquel circle.

(vi$a$) The limacons include the five cardioids as well.

(vii) The two common points lying on the Miquel circle (result vi) are equidistant from Z. Their joins form a system of parallel chords bisected by the diameter through Z (in theorem v).

(viii) For points of the Miquel circle, the eccentricity of the pedal conic is *stationary* for the point diametrically opposite to Z in (v).

(ix) If M_1, ... M_5 be the Miquel points of the lines taken four at a time and $M_1 T_1$, ... $M_5 T_5$ be the chords of the Miquel circles touching the respective centre circles, at M_1, ... M_5, then the lengths of the chords are proportional to $M_1 Z$, ... $M_5 Z$.

Vide *J. I. M. S.* Vol. XVI, p. 282.

GLEANINGS

48. *A poetic solution of the quadratic equation*

When you have written $-b$,
The double sign put down; then b^2-4ac
With square root mark you crown;
Beneath it all a line you trace,
Beneath which $2a$ you place.

From *The Mathematical Gazette*, Vol. 4.

49. THE A, B, C, OF THE HIGHER ANALYSIS

1. When Algebra, fair Daughter of king Arithmetic, is nearing the age of adolescence her eyes fall on Prince Limit, and there cometh a change upon her. Her tender heart goes forth to him and in her lithe and nimble frame she feels a torture from end to end as it would fall to pieces. She is in love and the Prince reciprocates her love.

2. Happily they are wedded. The king gave away his Daughter, the Angels witnessed it, and all was joy. The marriage was performed with the Hymn of the Transcendent Number and Ratio preceded by the Song of limits and followed by the Chorus of Continuity and Convergence.

3. Algebra is happy for evermore and with an Amplitude of vision she never knew before, and with her lord by her side, she surveys the vast Domains of Higher Analysis, wherein the king's Ancient laws prevail.

4. Three children are born of the happy union—all at one birth—and the glad news doth spread in Heaven. By the king's command Newton stood Sponsor and Leibnitz, of felicitous Power Symbolic conducted the Baptism. The ceremony was grand and the world witnessed it.

5. But soon the ceremony was over, the silly world started asking, which was the greater on the occasion the Minister or Sponsor? And they answered differently as their eyes were priestly or lay, and there was strife. Forgotten be this strife, for Behold the children have Eternal life.

6. Each of the princes, hath features derived from both his Royal parents and each also reveals the Imperial form of the king, yet each hath his own characteristics.

7. The first Prince is by habit an Astronomer. He delights in Infinitude. The depths of space and the mysteries thereof engage his attention. The Cometic Systems obey him, and have yielded him the Secret of their Tails. The king creates him Lord of the Infinite or Astronomical Branch of the Higher Analysis.

8. The Second Prince is a Biologist much after his Mother. He delights in the Evolution of Forms. He sees the life beneath every Form and thus the Secret of Form he knows and of that which changeth not with the Change of Form. The king creates him lord of the Finite or Biological Branch of the Higher Analysis.

9. The third Prince is ever a Chemist. The Atomic Structure of things delights him and he sees worlds within worlds in an Atom. He sees the atoms integrate and in Elemental Atomic Relations he discovers Forms and Form-Aggregates and Complexities thereof defying Thought. The king creates him lord of the Infinitesimal Branch in the Chemistry of the Higher Analysis.

10. Whoever says that he knoweth his A, B, C, thoroughly let him stand forward.

11. The little Princes love each other dearly as they do their Parents and their Grandfather, and have grown in each other's love sometimes playing each other tricks. And now they have gone forth together to conquer the Higher realms of Functions. Verily they shall have Eternal life and Eternal youth.

12. There be some who would set the Princes by the ears, trying to tickle one, and suggesting names to call another by. But the Princes shall not hear them for Newton is their Guardian.

Gooty, 7th March, 1907.

V. RAMASWAMI AIYAR in The Mathematical Gazette, pp. 79, Vol. IV.

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THE MATHEMATICAL ASSOCIATION OF AMERICA

The Mathematical Association of America is devoted primarily to the interests of collegiate mathematics. It now has over two thousand individual and institutional members. There are twenty sections at present, representing thirty-one different States. The Association holds two national meetings per year and the sections hold one or two meetings per year. All meetings both national and sectional, are reported in the official Journal.

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Remittances of membership fees and of subscriptions should be sent to the Treasurer, Prof. L. N. SUBRAMANIAM, Department of Mathematics, Christian College, Madras.